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Optimal Minimum Wage Policy in Competitive Labor Markets*

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Abstract

This paper provides a theoretical analysis of optimal minimum wage policy in a perfectly competitive labor market. We show that a binding minimum wage – while leading to unemployment – is nevertheless desirable if the government values redistribution toward low wage workers and if unemployment induced by the minimum wage hits the lowest surplus workers first. This result remains true in the presence of optimal nonlinear taxes and transfers. In that context, a minimum wage effectively rations the low skilled labor that is subsidized by the optimal tax/transfer system, and improves upon the second-best tax/transfer optimum. When labor supply responses are along the extensive margin, a minimum wage and low skill work subsidies are complementary policies; therefore, the co-existence of a minimum wage with a positive tax rate for low skill work is always (second-best) Pareto inefficient. We derive formulas for the optimal minimum wage (with and without optimal taxes) as a function of labor supply and demand elasticities and the redistributive tastes of the government. We also present some illustrative numerical simulations.

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1 Introduction

The minimum wage is a widely used but controversial policy tool. Although a potentially useful tool for redistribution because it increases low skilled workers' wages at the expense of other factors of production (such as higher skilled workers or capital), it may also lead to involuntary unemployment, thereby worsening the welfare of workers who lose their jobs. An enormous empirical literature has studied the extent to which the minimum wage affects the wages and employment of low skilled workers.[1] The normative literature on the minimum wage, however, is much less extensive.

This paper provides a normative analysis of optimal minimum wage in a conventional competitive labor market model, using the standard social welfare framework adopted in the optimal tax theory literature following the seminal contributions of Diamond and Mirrlees (1971) and Mirrlees (1971). In most of our analysis, we adopt the important "efficient rationing" assumption – that unemployment induced by the minimum wage hits workers with the lowest surplus first.[2] Our goal is to use this framework to illuminate the trade-offs involved when a government sets a minimum wage, and to shed light on the appropriateness of a minimum wage in the presence of optimal taxes and transfers.

The first part of the paper considers a competitive labor market with no taxes/transfers. Although unrealistic, this case illustrates the key trade-off when choosing a minimum wage rate.[3] We show that a binding minimum wage is desirable as long as the government places a non-zero value on redistribution from high- to low-wage workers, the demand elasticity of low skilled labor is finite, and the supply elasticity of low skilled labor is positive. Unsurprisingly, the resulting optimal minimum wage is decreasing in the demand elasticity because a minimum wage has larger unemployment effects when the demand elasticity is higher. The optimal minimum wage is increasing in the supply elasticity because a high supply elasticity implies that marginal workers have a low surplus from working (since many would leave the labor force if the wages were slightly reduced). The size of the optimal minimum wage follows an

[1] See e.g., Brown et al. (1982), Card and Krueger (1995), Dolado et al. (1996), Brown (1999), or Neumark and Wascher (2006) for extensive surveys.

[2] Although we believe that efficient rationing is the most natural assumption, we also discuss in detail how our results are modified if unemployment hits low skilled workers independently of surplus, what we call "uniform rationing".

[3] Although simple, this analysis does not seem to have been formally derived in the previous literature.

inverted U-shape with the degree of the government's redistributive tastes: there is no role for the minimum wage if the government neither values redistribution nor has extreme Rawlsian preferences (as the costs of involuntary unemployment dominate the value of transfers to low skilled workers).

The second part of the paper considers how the results change when the government also uses taxes and transfers to achieve redistributive goals. As described below, our key innovation is to abstract from the hours of work decision and focus only on the job choice and work participation decisions. In that context, the government observes only occupation choices and corresponding wages, but not the utility work costs incurred by individuals. Therefore, the informational constraints the government faces when imposing a minimum wage policy and a nonlinear tax/transfer system are well defined and mutually consistent. In such a model, we show that a minimum wage is desirable if rationing is efficient and the government values redistribution toward low skilled workers. This result can be seen as an application of the Guesnerie (1981) and Guesnerie and Roberts (1984) theory of quantity controls in second best economies: when the government values redistribution toward low skilled workers, the optimal tax/transfer system over-encourages the supply of low skilled labor. In that context, a minimum wage effectively rations over-supplied low skilled labor, which is socially desirable. In other words, if the minimum wage rations low skilled jobs, the government can increase redistribution toward those workers without inducing any adverse supply response. Theoretically, the minimum wage under efficient rationing sorts individuals into employment and unemployment based on their unobservable cost of work. Thus, the minimum wage partially reveals costs of work in a way that tax/transfer systems cannot.[4]

When labor supply responses are along the participation margin, we show that a minimum wage should always be associated with work subsidies (such as the US Earned Income Tax Credit). Consequently, imposing positive tax rates on the earnings of minimum wage workers is second-best Pareto inefficient: cutting taxes on low income workers while reducing the (pre-tax) minimum wage leads to a Pareto improvement. This result remains true even if rationing is inefficient and could be widely applied in many OECD countries with significant minimum wages and high tax rates on low skilled work.

[4]Unsurprisingly, we show that if rationing is uniform (and hence does not reveal anything on costs of work), then the minimum wage cannot improve upon the optimal tax/transfer allocation.

We derive formulas for the jointly optimal tax/transfer system and minimum wage. The formulas, as well as numerical simulations, show that – as in the basic case without taxes and transfers – the optimal minimum wage with optimal taxes is again decreasing in the demand elasticity for low skilled work, increasing in the supply elasticity for low skilled work, and it follows an inverted U-shape pattern with respect to the strength of redistributive tastes.

The remainder of the paper is organized as follows. Section 2 provides an overview of the existing literature most relevant to our analysis. Section 3 presents the basic two-skill model with extensive labor supply responses and analyzes optimal minimum wage policy with no taxes. Section 4 introduces taxes and transfers and analyzes jointly optimal minimum wage policy and taxes/transfers. Section 5 presents illustrative numerical simulations. Section 6 briefly concludes. Formal technical proofs of our propositions are presented in Appendix A, while Appendix B contains several extensions such as "uniform rationing" and more general labor supply responses.

2 Existing Literature

That a large demand elasticity for low skilled workers implies a large negative employment effect of minimum wage will be large has been recognized for a long time (see e.g. Pigou, 1920 and Stigler, 1946). A well-known related point is that, if the absolute value of the demand elasticity is greater than one, the minimum wage reduces the total pay to low skilled workers (see e.g. Freeman, 1996; Dolado, Felgueroso, and Jimeno, 2000). In contrast, our analysis reveals no special significance to the absolute demand elasticity being one, but highlights the importance of labor supply elasticities. We can divide the recent normative literature the minimum wage into two strands.

The first, most closely associated with labor economics, focuses on efficiency effects of the minimum wage in the presence of labor market imperfections. It is well known, at least since Robinson (1933), that if the labor market is monopsonistic, a minimum wage can increase both employment and low skilled wages therefore improving efficiency (see e.g., Card and Krueger, 1995 or Manning, 2003 for recent expositions). A number of papers have shown that the monopsony logic for the desirability of the minimum wage extends to other models of the labor market with frictions or informational asymmetries such as efficiency wages (Drazen, 1986, Jones, 1987, Rebitzer and Taylor, 1995), bargaining models (Cahuc, Zylberberg, and Saint-Martin, 2001), signalling models (Lang, 1987), search models (Swinnerton, 1996, Acemoglu 2001, Flinn, 2006), Keynesian macro models (Foellmi and Zweimuller, 2007), or endogenous growth models (Cahuc and Michel, 1996). These studies focus on efficiency and generally abstract from the government's redistributive goals. They do not consider the minimum wage when taxes and transfers are available to achieve these goals.

A second smaller literature in public economics investigates whether the minimum wage is desirable for redistributive reasons in situations where the government can also use optimal taxes and transfers for redistribution. The general principle, following Allen (1987) and Guesnerie and Roberts (1987), is that a minimum wage is desirable if it expands the redistributive power of the government by relaxing incentive compatibility constraints. In the context of the two-skill Stiglitz (1982) model with endogenous wages, Allen (1987) and Guesnerie and Roberts (1987) show that a minimum wage can sometimes usefully supplement an optimal *linear* tax,[5] but is never useful in the presence of an optimal nonlinear tax even in the most favorable case where unemployment is efficiently shared. This result is obtained because a minimum wage does not in any way prevent high skilled workers from imitating low skilled workers in the Stiglitz (1982) model. This contrasts with our occupational model and we will return to this important difference.[6] By contrast, Boadway and Cuff (2001), using a continuum of skills model as in Mirrlees (1971), show that a minimum wage policy combined with forcing non-working welfare recipients to look for jobs and accept job offers indirectly reveals skills at the bottom of the distribution. This can be exploited by the government to target welfare on low skilled individuals, thus improving upon the standard Mirrlees (1971) allocation.[7]

As recognized by Guesnerie and Roberts (1987), these contrasting results stem in part from informational inconsistencies that arise when a minimum wage is introduced: the minimum wage implementation requires observing wage rates, while the income tax is based on earnings (because it is assumed that awge rates and hours of work are not separately observable for

[5]Allen (1987) notes, consistently with our results, that the minimum wage is more likely to be desirable when the labor supply elasticity is high.

[6]Marceau and Boadway (1994) build upon those papers and show that a minimum wage can be desirable when a participation constraint for low skilled workers is introduced. Although Marceau and Boadway do not explicitly model this participation constraint using fixed costs of work as we do, their paper can be seen as a first step in incorporating the labor force participation decision in the problem.

[7]Remarkably, this result is obtained in a fixed wage model where the minimum wage destroys all jobs below the minimum wage.

tax purposes). If wage rates are directly observable, the government can achieve any first best allocation by conditioning taxes and transfers on immutable wage rates (and obviously, no minimum wage would be needed). The negative results on the desirability of the minimum wage of Allen (1987) appear in an environment where the government implicitly observes the wage rates for low skilled workers – a necessity when implementing a minimum wage – yet ignores this extra information when choosing the income tax. On the other hand, the positive results of Boadway and Cuff (2001) are obtained because the government uses other tools that implicitly exploit information revealed by the minimum wage.[8] Our analysis resolves this informational inconsistency by abstracting from the hours of work decision and focusing only on job choice and work participation decisions.[9]

Finally, some recent studies have brought together those two literature strands and explored the issue of jointly optimal minimum wages and optimal taxes and transfers in imperfect labor markets. Blumkin and Sadka (2005) consider a signalling model where employers do not observe productivities perfectly and show that a minimum wage can be desirable to supplement the optimal tax system. Cahuc and Laroque (2007) show that, in a monopsonistic labor market model, with participation labor supply responses only, the minimum wage should not be used when the government can use optimal nonlinear income taxation. Hungerbuhler and Lehmann (2007) analyze a search model and show that a minimum wage can improve welfare even with optimal income taxes if the bargaining power of workers is sufficiently low. There, however, if the government can directly increase the bargaining power of workers, the desirability of the minimum wage vanishes. These latter two papers are most similar to our analysis in the sense that they also abstract from the hours of work choice and consider only the participation margin for labor supply. Our analysis, however, considers the simple case of perfect competition with no market frictions. Therefore, we see our contribution as complementary to those of Cahuc and Laroque (2007) and Hungerbuhler and Lehmann (2007).

[8] Some papers have actually explicitly modelled limitations on the use of taxes and transfers using political economy arguments. In that context, a minimum wage can be a useful tool for redistribution (see e.g., Drèze and Gollier, 1993 and Bacache and Lehmann, 2005).

[9] Although informational consistency is conceptually appealing, governments do use minimum wages based on hours of work and income taxes based on earnings. Hence, it is still useful to consider the constrained optimization problem combining taxes on earnings and minimum wage rates. Therefore, we will explain in greater detail the deeper economic reasons why our results differ from those of Allen (1987).

3 Optimal Minimum Wage with no Taxes/Transfers

3.1 The Model

• Demand Side

We consider a simple model with two labor inputs where production of a unique consumption good $F(h_1, h_2)$ depends on the number of low skilled workers h_1 and the number of high skilled workers h_2. We assume perfectly competitive markets so that firms take wages (w_1, w_2) as given. The production sector chooses labor demand (h_1, h_2) to maximize profits: $\Pi = F(h_1, h_2) - w_1 h_1 - w_2 h_2$, which leads to the standard first order conditions where wages are equal to marginal product:

$$w_i = \frac{\partial F}{\partial h_i}, \tag{1}$$

for $i = 1, 2$. We assume that in any equilibrium $w_1 < w_2$. We also assume constant returns to scale, so that there are no profits in equilibrium: $\Pi = F(h_1, h_2) - w_1 h_1 - w_2 h_2 = 0$.

• Supply Side

We assume each individual is either low skilled or high skilled. We normalize the population of workers to one and denote by h_1^0 and h_2^0 the fraction of low and high skilled with $h_1^0 + h_2^0 = 1$. Each worker faces a cost of working, θ, representing her disutility of work. In order to generate smooth supply curves, we assume that θ is distributed according to smooth cumulative distributions $P_1(\theta)$ and $P_2(\theta)$ for low and high skilled individuals respectively. There are three groups of individuals: group 0 for unemployed individuals (either low or high skilled) with zero earnings, group 1 for low skilled workers earning w_1, and group 2 for high skilled workers earning w_2. We denote by h_i the fraction of individuals in each group $i = 0, 1, 2$.

In this section, we assume that there are no taxes/transfers. To simplify the exposition, throughout the paper, we assume no income effects in the labor supply decision.[10] An individual with skill i and cost of work θ makes her binary labor supply decision $l = 0, 1$ to maximize utility $u = w_i \cdot l - \theta \cdot l$. Therefore, $l = 1$ if and only if $\theta \leq w_i$. Hence, the aggregate labor supply functions for $i = 1, 2$ are:

$$h_i = h_i^0 \cdot P_i(w_i). \tag{2}$$

[10] The presence of income effects would not change our key results as we show in Appendix B.3.

We denote by e_i the elasticity of labor supply h_i with respect to the wage w_i:

$$e_i = \frac{w_i}{h_i}\frac{\partial h_i}{\partial w_i} = \frac{w_i \cdot p_i(w_i)}{P_i(w_i)},$$

where $p_i = P_i'$ is the density distribution of θ.

• Competitive Equilibrium and Labor Demand

Combining the demand and supply side equations (1) and (2) defines a single undistorted competitive equilibrium denoted by $(w_1^*, w_2^*, h_1^*, h_2^*)$.

Figure 1a shows the competitive equilibrium for low skilled labor using standard supply and demand curve representation. The supply curve is defined as $h_1 = h_1^0 P_1(w_1)$. Due to constant returns to scale in production, only the ratio h_1/h_2 is well defined on the demand side. For our purposes, we define the demand for low skilled work $h_1 = D_1(w_1)$ as follows: $D_1(w_1)$ is the level of demand when w_1 is set exogenously by the government (such as with a minimum wage policy) and (h_2, w_2) is defined as the market clearing equilibrium on the high skilled labor market. Therefore, Figure 1a implicitly captures general equilibrium effects as well.[11] The low skilled labor demand elasticity η_1 is defined as:

$$\eta_1 = -\frac{w_1}{h_1} \cdot D_1'(w_1), \tag{3}$$

where the minus sign normalization is used so that $\eta_1 > 0$.

• Government Social Welfare Objective

We assume that the government evaluates outcomes using a standard social welfare function of the form: $SW = \int G(u)d\nu$ where $u \to G(u)$ is an increasing and concave transformation of the individual money metric of individual utilities $u = w_i - \theta \cdot l$. The concavity of $G(.)$ represents either individuals' decreasing marginal utility of money and/or the redistributive tastes of the government. Given the structure of our model, we can write social welfare as:

$$SW = (1 - h_1 - h_2)G(0) + h_1^0 \int G(w_1 - \theta)p_1(\theta)d\theta + h_2^0 \int_0^{w_2} G(w_2 - \theta)p_2(\theta)d\theta. \tag{4}$$

[11]For example, in the case of a CES production function $F(h_1, h_2) = (a_1 h_1^{(\sigma-1)/\sigma} + a_2 h_2^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}$, the ratio of the demand side equations (1) implies that $h_1 = h_2 \cdot (a_1/a_2)^\sigma \cdot (w_2/w_1)^\sigma$. The no profit condition $F = w_1 h_1 + w_2 h_2$ implies that $a_1^\sigma w_1^{1-\sigma} + a_2^\sigma w_2^{1-\sigma} = 1$, which defines $w_2(w_1)$ as a function of w_1. The supply equation $h_2 = h_2^0 P_2(w_2)$ then defines $h_2(w_1)$ as a function w_1. Therefore, we have $D_1(w_1) = h_2(w_1) \cdot (a_1/a_2)^\sigma \cdot (w_2(w_1)/w_1)^\sigma$.

With no minimum wage, integration in the second term of (4) goes from $\theta = 0$ to w_1 but not when a minimum wage is binding, as we will discuss below. It is useful for our analysis to introduce the concept of social marginal welfare weights at each occupation. Formally, we define $g_0 = G'(0)/\lambda$ and $g_i = h_i^0 \int G'(w_i - \theta) p_i d\theta / (\lambda \cdot h_i)$ as the average social marginal welfare weight of individuals in occupation $i = 1, 2$. The normalization factor $\lambda > 0$ is chosen so that those weights average to one: $h_0 g_0 + h_1 g_1 + h_2 g_2 = 1$.[12] Intuitively, g_i measures the social marginal value of redistributing one dollar uniformly across all individuals in occupation i. In our model, because individuals cannot be forced to work, workers are better off than non-workers. Hence concavity of $G(.)$ implies $g_0 > g_1$ and $g_0 > g_2$.

3.2 Desirability of the Minimum Wage

Starting from the market equilibrium $(w_1^*, w_2^*, h_1^*, h_2^*)$, and illustrated in Figure 1a, we introduce a *small* minimum wage just above the low skilled wage w_1^*, which we denote by $\bar{w} = w_1^* + d\bar{w}$. The small minimum wage creates changes dw_1, dw_2, dh_1, dh_2 in our key variables of interest. By definition, $dw_1 = d\bar{w}$. From $\Pi = F(h_1, h_2) - w_1 h_1 - w_2 h_2$, we have $d\Pi = \sum_i [(\partial F/\partial h_i) dh_i - w_i dh_i - h_i dw_i] = -h_1 dw_1 - h_2 dw_2$ using (1). The no profit condition $\Pi = 0$ then implies $d\Pi = 0$ and hence:

$$h_1 dw_1 + h_2 dw_2 = 0. \tag{5}$$

Equation (5) is fundamental and shows that the earnings gain of low skilled workers $h_1 dw_1 > 0$ (the dark red dashed rectangle on Figure 1a) due to a small minimum wage is entirely compensated by an earnings loss of high skilled workers $h_2 dw_2 < 0$. If $g_2 < g_1$ (i.e., the government values redistribution from high skilled workers to low skilled workers) such a transfer is socially desirable.

However, in addition to this transfer, the minimum wage also creates involuntary unemployment (also depicted in Figure 1a). To evaluate the welfare cost of the involuntary unemployment, we will make the important assumption of *efficient rationing.*

Assumption 1 ***Efficient Rationing**: Workers who involuntarily lose their jobs due to the minimum wage are those with the least surplus from working.*

[12]In Section 4, we will show that λ is naturally the multiplier of the government budget constraint when the government uses taxes and transfers.

Conceptually, the minimum wage creates involuntary unemployment and hence an allocation problem: which workers become involuntarily unemployed due to the minimum wage? Under costless Coasian bargaining, this allocation problem would be resolved efficiently: a worker with a low surplus from working would be willing to let an unemployed worker with a high surplus take her job in exchange for a private transfer, leading to efficient rationing overall. In practice, the efficient allocation might be reached because workers with the least surplus are more likely to quit through natural attrition and because, if turnover is costly, employers may first lay off workers who are least likely to be stable employees (i.e., those with low surplus from the job).[13]

In the end, determining which workers lose their jobs due to the minimum wage is an empirical question. Unfortunately, empirical work on this question is thin. In the United States, evidence of unemployment effects is stronger among teenagers and secondary earners (Neumark and Wascher 2006) who are likely to be more elastic - and hence have a lower surplus - suggesting that rationing might be efficient. More directly, Luttmer (2007) used variation in state minimum wages to show (proxies for) reservation wages do not increase following an increase in the minimum wage, suggesting that minimum wage induced rationing is efficient.[14] Obviously, the case with efficient rationing is the most favorable to minimum wage policy. Therefore, in Appendix B.1 we also explore how our results change if we assume that unemployment losses are distributed independently of surplus.

Under efficient rationing, as can be seen in Figure 1a, as long as the supply elasticity is positive (non-vertical supply curve) and the demand elasticity is finite (non-horizontal demand curve), those who lose their jobs because of $d\bar{w}$ have infinitesimal surplus. Therefore, the welfare loss due to involuntary unemployment caused by the minimum wage is second order and represented by the dashed light green triangle (exactly as in the standard Harberger deadweight burden analysis). As a result, we have:

Proposition 1 *With no taxes/transfers and under Assumption 1 (efficient rationing), introducing a minimum wage is desirable if (1) the government values redistribution from high skilled workers toward low skilled workers ($g_1 > g_2$); (2) the demand elasticity for low skilled*

[13]It is conceivable, however, that resources (such as search costs or queuing costs) could be dissipated in reaching the efficient allocation.

[14]This is in contrast to a situation with low turnover, such as in the housing market with rent control, as in Glaeser and Luttmer (2003).

workers is finite; and (3) the supply elasticity of low skilled workers is positive.

The formal proof is presented in Appendix A.1. It is useful to briefly analyze the desirability of the minimum wage when any of those three conditions does not hold. Condition (1) is necessary: it obviously fails if the government does not care about redistribution at all ($g_1 = g_2$). It also fails in the extreme case where the government has Rawlsian preferences and only cares about those out of work, meaning it values the marginal income of low and high skilled workers equally ($g_1 = g_2 = 0$). Therefore, a minimum wage is desirable only for intermediate redistributive tastes. Even in that case, condition (1) may fail if minimum wage workers actually belong to well-off families (for example teenagers or secondary earners).[15]

Condition (2) is also necessary. If the demand elasticity is infinite, which in our model is equivalent to assuming low and high skill workers are perfect substitutes, (so that $F = a_1 h_1 + a_2 h_2$ with fixed parameters a_1, a_2), then any minimum wage set above the competitive wage $w_1^* = a_1$ will completely shut down the low skilled labor market and therefore cannot be desirable. A large body of empirical work suggests that the demand elasticity for low skilled labor is not infinite (see e.g. Hamermesh, 1996 for a survey). In addition, evidence of a spike in the wage density distribution at the minimum wage also implies a finite demand elasticity (Card and Krueger, 1995).

When condition (3) breaks down and the supply elasticity is zero, then there are no marginal workers with zero surplus from working. Therefore, the unemployment welfare loss is no longer second order. In that context, whether a minimum wage is desirable depends on the parameters of the model (specifically, the reservation wages of low skilled workers and the size of demand elasticity).[16] Empirically, a large body of work has shown that there are substantial participation supply elasticities for low skilled workers (see e.g., Blundell and MaCurdy, 1999 for a survey).

Finally, as we show in Appendix B.1, if the efficient rationing assumption is replaced by uniform rationing (i.e., unemployment strikes independently of surplus), then a small minimum wage creates a first order welfare loss. In that case, a minimum wage may or may not be

[15]It would be straightforward to capture such an effect in our model by assuming that utility depends also on other household members income. We would simply need to adjust the social welfare weights g_i accordingly. Kniesner (1981), Johnson and Browning (1983) and Burkhauser, Couch, and Glenn (1996) empirically analyze this issue in the United States.

[16]The well known result that a minimum wage cannot be desirable if $\eta_1 > 1$ is based on such a model with fixed labor supply.

desirable depending on the parameters of the model.

3.3 Optimal Minimum Wage

Let us now derive the optimal minimum wage when the conditions of Proposition 1 are met. As displayed in Figure 1b, with a non infinitesimal minimum wage $\bar{w} > w_1^*$, we can define $\underline{w}$ as the reservation wage (or equivalently, the cost of work) of the marginal low skilled worker (i.e. the worker getting the smallest surplus from working). Formally, $\underline{w}$ is defined so that $h_1^0 P_1(\underline{w}) = D_1(\bar{w})$. The government picks $\bar{w}$ to maximize

$$SW = (1 - D_1(\bar{w}) - h_2)G(0) + h_1^0 \int_0^{\underline{w}} G(w_1 - \theta)p_1(\theta)d\theta + h_2^0 \int_0^{w_2} G(w_2 - \theta)p_2(\theta)d\theta, \quad (6)$$

subject to the constraints that $w_i = \partial F / \partial h_i$ for $i = 1, 2$, the no profit condition $h_1 w_1 + h_2 w_2 = F(h_1, h_2)$, and $h_2 = h_2^0 P_2(w_2)$. This maximization problem is formally solved in Appendix A.1.

In order to obtain an intuitive understanding of the first order condition for the optimal minimum wage $\bar{w}$, we consider a small change $d\bar{w}$ around $\bar{w}$. Figure 1b shows that this change has two effects.

First, it creates a transfer $h_1 d\bar{w}$ toward low skilled workers at the expense of high skilled workers (as $h_2 dw_2 = -h_1 d\bar{w}$ from the no-profit condition (5)). Using the definition of g_i introduced earlier, the net social value of this transfer is $dT = [g_1 - g_2] h_1 d\bar{w}$.

Second, the minimum wage increases involuntary unemployment by $dh_1 = D_1'(\bar{w}) d\bar{w} = -\eta_1 h_1 d\bar{w} / \bar{w}$. Using the efficient rationing assumption, those marginal workers have a reservation wage equal to $\underline{w}$. Therefore, each newly unemployed worker has a social welfare cost equal to $G(\bar{w} - \underline{w}) - G(0)$. We can define $g_0^e = [G(\bar{w} - \underline{w}) - G(0)] / [\lambda \cdot (\bar{w} - \underline{w})]$ as the marginal welfare weight put on earnings lost due to unemployment. Thus, the welfare cost due to unemployment is $dU = -g_0^e \cdot (\bar{w} - \underline{w}) \cdot \eta_1 \cdot h_1 d\bar{w} / \bar{w}$.

Note that the change $dh_2 < 0$ does not generate welfare effects because marginal workers in the high skill sector have no surplus from working, making the welfare cost second order. At the optimum, we have $dT + dU = 0$, which implies:

$$\frac{\bar{w} - \underline{w}}{\bar{w}} = \frac{g_1 - g_2}{\eta_1 \cdot g_0^e}. \quad (7)$$

Formula (7) shows that the optimal minimum wage wedge (defined as $(\bar{w} - \underline{w})/\bar{w}$) is decreasing in the labor demand elasticity η_1 as a higher elasticity creates larger negative unemployment effects. The optimal wedge is increasing with $g_1 - g_2$, which measures the net value of transferring \$1 from high to low skilled workers, and decreasing in g_0^e, which measures the social cost of earning losses due to involuntary unemployment. Obviously g_0^e, g_1, and g_2 are endogenous parameters and depend on the primitive social welfare function $G(.)$ and also on the level of the minimum wage. At the optimum, however, we have $g_0^e \geq g_1 \geq g_2$. Increasing the redistributive tastes of the government by choosing a more concave $G(.)$ will have an ambiguous effect on the level of the optimal $\bar{w}$ because it will likely increase both $g_1 - g_2$ and g_0^e. As discussed above, the minimum wage should not be used if the government does not value redistribution at all ($g_1 = g_2$) or if the government has extreme Rawlsian tastes ($g_1 = g_2 = 0$). Therefore, we can expect the level of the optimal $\bar{w}$ to follow an inverted U-shape with the level of redistributive tastes.

Formula (7) is not an explicit formula because it depends on $\underline{w}$, which itself depends on $\bar{w}$ through the supply function (as illustrated on Figure 1b). However, if we assume that the elasticities of demand η_1 and supply e_1 are constant, then we can obtain explicit formulas. In this case $D_1(w_1) = D_0 \cdot w_1^{-\eta_1}$ and $S_1(w_1) = S_0 \cdot w_1^{e_1}$ so that $S_0 \cdot w_1^{*e_1} = D_0 \cdot w_1^{*-\eta_1}$ and $S_0 \cdot \underline{w}^{e_1} = D_0 \cdot \bar{w}^{-\eta_1}$. This implies that $\underline{w} = w_1^* \cdot (w_1^*/\bar{w})^{\eta_1/e_1}$, and hence:

$$\frac{\bar{w} - \underline{w}}{\bar{w}} = 1 - \left(\frac{w_1^*}{\bar{w}}\right)^{1+\frac{\eta_1}{e_1}}.$$

Formula (7) can thus be rewritten as:

$$\frac{\bar{w}}{w_1^*} = \left(1 - \frac{g_1 - g_2}{g_0^e \cdot \eta_1}\right)^{-\frac{e_1}{e_1+\eta_1}} \simeq 1 + \frac{e_1}{e_1 + \eta_1} \cdot \frac{g_1 - g_2}{g_0^e \cdot \eta_1}, \tag{8}$$

where the approximation holds in the case of a small minimum wage (i.e., when $(g_2 - g_1)/(g_0^e \cdot \eta_1)$ is small). The formula shows that the optimal minimum wage $\bar{w}$ is decreasing in the supply elasticity e_1. The intuition here can be easily understood from Figure 1b. A higher supply elasticity implies a flatter supply curve, and hence lower costs from involuntary unemployment. If the supply elasticity is high, then a small change in w_1 has large effects on supply, implying that workers derive little surplus from working and do not lose much from minimum wage induced unemployment. This result is very important because – as is well known – redistribution through taxes/transfers is hampered by a high supply elasticity. Conversely, when the supply elasticity is low, redistribution through minimum wage is costly while redistribution through taxes/transfers is efficient.

Formula (8) shows that there are two channels through which a higher demand elasticity η_1 reduces the optimal minimum wage. The first channel is the standard *unemployment level* effect mentioned when discussing (7), that a higher demand elasticity creates a larger unemployment response to the minimum wage. The second channel is an *unemployment cost* effect which works through the link between the wedge $(\bar{w} - \underline{w})/\bar{w}$ and the minimum wage markup $\bar{w}/w_1^*$. A higher demand elasticity implies that a given minimum wage markup is associated with a larger wedge, hence higher unemployment costs for the marginal worker. The distinction between those two channels is important because, as we will see later, the first classical unemployment level effect disappears with optimal taxes and transfers, but the unemployment cost effect remains.

The logic of our optimal minimum wage formula easily extends to a more general model with many labor inputs (including a continuum with a smooth wage density), a capital input or pure profits, and many consumption goods. In those contexts, g_2 is the average social welfare weight across each factor bearing the incidence of the minimum wage increase. Some of the factors can have a negative weight in this average. For example, if there are neo-classical spillovers of a minimum wage increase to slightly higher paid workers (as in Teulings, 2000), it is conceivable that g_2 could be negative. Conversely, if a minimum wage increase leads to higher consumption prices for goods consumed by low income families (such as fast food), g_2 would be higher (and conceivably even above g_1 if minimum wage workers belong to families with higher incomes than typical fast food consumers).

4 Optimal Minimum Wage with Taxes and Transfers

4.1 Introducing Taxes and Transfers

We assume that the government can observe job outcomes (not working, work in sector 1 paying w_1, or work in sector 2 paying w_2), but not the costs of work. Therefore, the government can condition tax and transfers only on observable work outcomes. Let us denote the tax on occupation i by T_i; T_i is a transfer if $T_i < 0$. We denote by $c_i = w_i - T_i$ the disposable income in occupation $i = 0, 1, 2$. This represents a fully general nonlinear income tax on earnings.

As in our previous model without taxes, an individual with skill $i = 1, 2$ deciding to work earns w_i but increases his disposable by $c_i - c_0$. We can therefore define a tax rate τ_i on skill

i workers: $1 - \tau_i = (c_i - c_0)/w_i$. An individual of skill $i = 1, 2$ and with costs of work θ works if and only if $\theta \leq c_i - c_0 = (1 - \tau_i)w_i$. Hence, the aggregate labor supply functions for $i = 1, 2$ are:

$$h_i = h_i^0 \cdot P_i((1 - \tau_i)w_i) = h_i^0 \cdot P_i(c_i - c_0). \tag{9}$$

As above, we denote by e_i the elasticity of labor supply with respect to the net-of-tax wage rate $w_i(1 - \tau_i) = c_i - c_0$:

$$e_i = \frac{(1 - \tau_i)w_i}{h_i} \frac{\partial h_i}{\partial (1 - \tau_i)w_i} = \frac{(1 - \tau_i)w_i \cdot p_i((1 - \tau_i)w_i)}{P_i((1 - \tau_i)w_i)}.$$

The demand side of the economy is unchanged. For given parameters c_0, τ_1, τ_2 defining a tax and transfer system, the four equations (1) and (9) for $i = 1, 2$ define the competitive equilibrium $(h_1^*, h_2^*, w_1^*, w_2^*)$.

Assuming no exogenous spending requirement, the government budget constraint can be written as:[17]

$$h_0 c_0 + h_1 c_1 + h_2 c_2 \leq h_1 w_1 + h_2 w_2. \tag{10}$$

We denote by λ the multiplier of the government budget constraint.

4.2 Minimum Wage Desirability with Fixed Tax Rates

We first analyze how our previous analysis on the desirability of the minimum wage is affected by the presence of taxes and transfers assuming that τ_1, τ_2 are exogenously fixed and that the transfer c_0 adjusts automatically to meet the government budget constraint when a small minimum wage $\bar{w} = w_1^* + d\bar{w}$ is introduced. We assume that the minimum wage applies to wages before taxes and transfers.[18] This assumption does not affect the desirability of a minimum wage and is the most convenient convention.

Proposition 2 *With fixed tax rates τ_1, τ_2, under Assumption 1 (efficient rationing) and assuming $e_1 > 0$ and $\eta_1 < \infty$, introducing a minimum wage is desirable if and only if*

$$g_1 \cdot (1 - \tau_1) - g_2 \cdot (1 - \tau_2) + \tau_1 - \tau_2 - \tau_2 \cdot e_2 - \tau_1 \cdot \eta_1 > 0. \tag{11}$$

[17] None of our results would be changed if we assumed a positive exogenous spending requirement for the government.

[18] In practice, the legal minimum wage applies to wages net of employer payroll taxes, but before employee payroll taxes, income taxes, and transfers. $\bar{w}$ should be interpreted as the minimum wage *including* employer taxes.

The proof is presented in Appendix A.2.

When $\tau_1 = \tau_2 = 0$, equation (11) reduces to $g_1 - g_2 > 0$ (Proposition 1). Equation (11) shows that with taxes/transfers, introducing a minimum wage creates four fiscal effects that need to be taken into account in the welfare analysis: first, transferring one dollar pre-tax from high to low skilled workers through the minimum wage implies a \$ $(1 - \tau_1)$ post tax transfer to low skilled workers and a \$ $(1 - \tau_2)$ post tax loss to high skilled workers (captured by the factor $(1 - \tau_i)$ multiplying g_1 and g_2 in (11)). Second, such a transfer creates a direct net fiscal effect $\tau_1 - \tau_2$. Third, the reduction in w_2 leads to a supply effect further reducing taxes paid by the high skilled by $e_2 \cdot \tau_2$ per dollar transferred. Finally, involuntary unemployment also creates a tax loss equal to $-\tau_1 \cdot \eta_1$ per dollar transferred.[19]

It is important to note that a minimum wage cannot be replicated with taxes and transfers. Returning to Figure 1a – the case with no taxes – it is tempting to think that a small tax on low skilled workers creates the same wedge between supply and demand as the minimum wage. However, to replicate the minimum wage, this small tax should be rebated lump-sum to low skilled workers only. Obviously, if the tax is rebated to low skilled workers, those who dropped out of work because of the tax would want to come back to work. Without a rationing mechanism preventing this labor supply response, taxes and transfers cannot achieve the minimum wage allocation.

Cahuc and Laroque (2007) make the point that a minimum wage can be replicated by a knife-edge nonlinear income tax such that $T(w) = w$ for $0 < w < \bar{w}$ (as nobody would want to work in a job paying less than $\bar{w}$, employers would be forced to pay at least $\bar{w}$ to attract workers), and concluded that a minimum wage is redundant with a fully general nonlinear income tax. This argument is mathematically correct, but such a knife-edge income tax is effectively a minimum wage. Our model rules out such knife-edge income taxes because we consider tax rates that are occupation specific (rather than wage level specific). However, a fully general knife-edge income tax could not do better than the combination of our occupation specific tax rates combined with a minimum wage. Therefore, we think the definition of the tax and minimum wage tools we use is the most illuminating to understand the problem of

[19]Note that when low skilled work is subsidized ($\tau_1 < 0$), then the unemployment created by a small minimum wage creates a positive fiscal externality proportional to the demand elasticity η_1. In such a situation, introducing a minimum wage would actually be desirable even without redistributive tastes ($g_1 = g_2 = 1$) if $-\tau_1 \cdot \eta_1 > \tau_2 \cdot e_2$.

joint minimum wage and tax optimization.

4.3 Optimal Tax Formulas with no Minimum Wage

The government chooses c_0, c_1, c_2 in order to maximize social welfare

$$SW = (1 - h_1 - h_2)G(c_0) + h_1^0 \int_0^{c_1 - c_0} G(c_1 - \theta)p_1(\theta)d\theta + h_2^0 \int_0^{c_2 - c_0} G(c_2 - \theta)p_2(\theta)d\theta,$$

subject to the budget constraint (10) with multiplier λ. As shown in Appendix A.3, we have the following conditions at the optimum:

$$h_0 \cdot g_0 + h_1 \cdot g_1 + h_2 \cdot g_2 = 1, \tag{12}$$

$$\frac{\tau_i}{1 - \tau_i} = \frac{1 - g_i}{e_i}, \tag{13}$$

for $i = 1, 2$. Equation (12) implies that the average of marginal welfare weights across the three groups $i = 0, 1, 2$ is one. Indeed, the value of distributing one dollar to everybody is exactly the average marginal social weight, and the cost of distributing one dollar in terms of revenue lost is also one dollar (as we have assumed away income effects).[20]

Equation (13) can be understood from Figure 2a. Starting from an allocation (c_0, c_1, c_2), increasing c_1 by $dc_1 > 0$ leads to a positive direct welfare effect $h_1 g_1 dc_1 > 0$, a mechanical loss in tax revenue $-h_1 dc_1 < 0$, and a behavioral response increasing work by $dh_1 = dc_1 \cdot e_1 h_1/(w_1(1 - \tau_1)) > 0$ and creating a fiscal effect equal to $\tau_1 w_1 dh_1 = dc_1 \cdot h_1 \cdot e_1 \cdot \tau_1/(1 - \tau_1)$. The sum of those three effects is zero, which implies (13).

If $g_1 > 1$, then the optimal tax rate on low skilled work should be negative because the first two terms net out positive so that the fiscal effect due to the behavioral response has to be negative, requiring $\tau_1 < 0$.[21]

Equations (12) and (13) are identical to those derived by Saez (2002) in the same model, but with fixed wages. Indeed, it is well known since Diamond and Mirrlees (1971), that optimal tax formulas remain the same when producer prices are endogenous.[22] Figure 2b illustrates this key point for our subsequent analysis. When w_1, w_2 are endogenous, the small reform

[20] See Appendix B.3. for an analysis with income effects.

[21] This was the key result emphasized by Diamond (1980), Saez (2002), Laroque (2005), Choné and Laroque (2005, 2006): an EITC type transfer for low wage workers is optimal in a situation where individuals respond only along the extensive margin.

[22] Piketty (1997) and Saez (2004) have shown that the occupational model we consider inherits this important property of the Diamond and Mirrlees (1971) model.

dc_1 leads to changes in h_1 and hence to changes dw_1 and dw_2 through demand side effects. However, assuming that c_2 and $c_1 + dc_1$ are kept unchanged, the effect of dw_1 and dw_2 is fiscally neutral because $h_1 dw_1 + h_2 dw_2 = 0$, which follows from the no-profit condition (5).

Let us denote by (w_i^T, c_i^T) the tax/transfer optimum with no minimum wage.

4.4 Optimal Minimum Wage under Optimal Taxes and Transfers

• Minimum Wage Desirability with Optimal Taxes and Transfers

As illustrated on Figure 3, starting from the tax/transfer optimum (w_i^T, c_i^T), let us introduce a minimum wage set at $\bar{w} = w_1^T$. Such a minimum wage is just binding and has no direct impact on the allocation. Let us now increase c_1 by dc_1 while keeping c_0 and c_2 constant. As we showed above, such a change provides incentives for some low skilled individuals to start working. However, as we showed in Figure 2b, such a labor supply response would reduce w_1 through demand side effects. However, in the presence of a minimum wage $\bar{w}$ set at w_1^T, w_1 cannot fall, implying that those individuals willing to start working cannot work and actually shift from voluntary to involuntary unemployment. The assumption of efficient rationing is key here as these are precisely the individuals with the lowest surplus from working. Given that the labor supply channel is effectively shut down by the minimum wage, the dc_1 change is like a lump-sum tax reform and its net welfare effect is simply $[g_1 - 1]h_1 dc_1$. This implies that if $g_1 > 1$, introducing a minimum wage improves upon the tax/transfer optimum allocation.[23]

This result corresponds with the theory of optimum quantity controls developed by Guesnerie (1981) and Guesnerie and Roberts (1984) showing that, in an optimum Ramsey tax model, introducing a quantity control on subsidized goods is desirable. In our model, a minimum wage is an indirect way for the government to introduce rationing on low skilled workers subsidized by the optimal tax system.[24]

We show in Appendix B.2 this result generalizes easily to a broader model with many skills and fully general labor supply response functions where individuals can respond along the (discrete) intensive margin by shifting to lower paid occupations in response to taxes.

[23]The fact that a minimum wage is desirable if $g_1 > 1$ can also be seen from Proposition 2 by using the optimal tax rates from equations (13). In that case, equation (11) boils down to $-\tau_1 \cdot (e_1 + \eta_1) > 0$ which is indeed equivalent to $g_1 > 1$.

[24]Guesnerie and Roberts (1987) proposed an analysis of optimal minimum wage. However, the model they considered was not directly related to their earlier optimum quantity constraints theory (see our discussion just below).

The logic of the minimum wage desirability remains exactly the same as the one displayed in Figure 3: even if higher skilled workers wanted to shift to occupation w_1 when c_1 increases, a minimum wage set at w_1^T would effectively block such a labor supply response (again under our key assumption of efficient rationing).

This remark can help explain why our results contrast with the negative results of Allen (1987) or Guesnerie and Roberts (1987) obtained in the context of the Stiglitz (1982) two-type model of optimal nonlinear taxation. The key theoretical difference between the Stiglitz model and the occupation model we use is that in the Stiglitz model high skilled individuals imitating low skilled individuals cut their hours of work, but remain in the high skill sector. Thus the minimum wage makes it easier for them to imitate low skilled workers. In contrast, in our model the minimum wage effectively prevents high skilled workers from occupying minimum wage jobs (by rationing low skilled work). Perhaps more importantly, absent the minimum wage, everybody works in the Stiglitz model, which therefore cannot capture the participation decision of low skilled workers - a decision which strikes us as central to the minimum wage problem in the real world.[25]

Comparing with the case with no taxes in Section 3, we note that the condition $g_1 > 1$ is stronger than the earlier condition $g_1 > g_2$ (as g_0, g_1, g_2 average to one and $g_0 > g_1 > g_2$, we have $g_2 < 1$). However, if the government has redistributive tastes, then $g_1 > 1$ is a weak condition as the low skilled sector can be chosen to represent the very lowest income workers. This also implies that, when the government uses taxes optimally and in the presence of many factors of production or many output goods, the incidence of the minimum wage on other factors (captured by the term g_2 in the case with no taxes) becomes irrelevant: the government can effectively undo the incidence effects by adjusting taxes on other factors, keeping their net-of-tax rewards constant.[26] In particular, whether the minimum wage creates neo-classical spill-over effects on slightly higher wages and whether the minimum wage increases prices of goods disproportionately consumed by low income families are irrelevant when assessing the

[25] Indeed, Marceau and Boadway (1994) show that a minimum wage can be desirable in a Stiglitz type model by implicitly adding fixed costs of work (and hence a participation decision) for low skilled workers. Marceau and Boadway (1994) do not model explicitly fixed costs of work, but such fixed costs are necessary for the assumptions of their main proposition (p. 78) to be met. Our model has the advantage of explicitly modelling the participation decision and also avoiding the information inconsistency inherent to the Stiglitz model with minimum wage.

[26] This is directly related to the important fact that incidence on pre-tax prices is irrelevant in optimal Diamond-Mirrlees tax formulas.

desirability of the minimum wage in the presence of optimal taxes. The only relevant factor is whether the government values redistribution to minimum wage workers relative to an across the board lump-sum redistribution (i.e., the condition $g_1 > 1$).

Finally, we show in Appendix B.1 that the desirability of the minimum wage hinges crucially on the "efficient rationing" assumption. We show that, under "uniform rationing" (where unemployment strikes independently of surplus), the minimum wage cannot improve upon the optimal tax allocation. Indeed, with efficient rationing, a minimum wage effectively reveals the marginal workers to the government. Since costs of work are unobservable, this is valuable because it allows the government to sort workers into a more (socially albeit not privately) efficient set of occupations, making the minimum wage desirable. In contrast, with uniform rationing, a minimum wage does not reveal anything about costs of work (as unemployment strikes randomly). As a result, it only creates (privately) inefficient sorting across occupations without revealing anything of value to the government. It is not surprising that a minimum wages would not be desirable in this context.

• Optimal Minimum Wage with Taxes and Transfers

Let us now turn to the joint optimization of the tax/transfer system and the minimum wage. Formally, the government chooses $\bar{w}, c_0, c_1, c_2$ to maximize

$$SW = (1 - h_1 - h_2)G(c_0) + h_1^0 \int_0^{\underline{w}(1-\tau_1)} G(c_1 - \theta)p_1(\theta)d\theta + h_2^0 \int_0^{c_2 - c_0} G(c_2 - \theta)p_2(\theta)d\theta. \quad (14)$$

subject to its budget constraint (with multiplier λ). As above, $\underline{w}$ is defined as the reservation wage of the marginal worker: $h_1^0 \cdot P_1(\underline{w}(1 - \tau_1)) = D_1(\bar{w})$ where $D_1(\bar{w})$ is the demand for low skilled labor for a given minimum wage $\bar{w}$. The second term in (14) incorporates the efficient rationing assumption as workers are those with the lowest cost of work and hence the highest surplus.

We solve this maximization problem formally in Appendix A.4. The first order condition with respect to c_0 implies that $h_0 g_0 + h_1 g_1 + h_2 g_2 = 1$. The first order condition with respect to c_2 leads to the standard formula (13): $\tau_2/(1 - \tau_2) = (1 - g_2)/e_2$, as the minimum wage does not impact the trade-off for the choice of c_2.

With a binding minimum wage, as we illustrated in Figure 3, increasing c_1 is a lump-sum transfer. Therefore, the government will increase c_1 up to the point where $g_1 = 1$. A minimum wage allows the government to redistribute to low skilled workers at no efficiency cost and hence achieve "full redistribution to low skilled workers," making the minimum wage a powerful redistributive tool. We show in Appendix B.2 that this result is easily generalized to a model with numerous labor inputs and more general labor supply responses.

Finally, there is a first order condition for the optimal choice of $\bar{w}$. Increasing $\bar{w}$ by $d\bar{w}$ and keeping c_0, c_1, c_2 constant leads to an increase in involuntary unemployment: $dh_1 < 0$. Such involuntary unemployment leads to a (negative) welfare effect on those individuals equal to $dh_1[G(c_0 + (\bar{w} - \underline{w})(1-\tau_1)) - G(c_0)]/\lambda < 0$ and a fiscal effect equal to $dh_1 \cdot \tau_1 \cdot \bar{w}$.[27] Therefore, the two effects caused by dh_1 need to cancel out at the optimum. Hence the fiscal effect needs to be positive, requiring $\tau_1 < 0$ as $dh_1 < 0$. We then have the following first order condition:

$$-\tau_1 \cdot \bar{w} = \frac{G(c_0 + (\bar{w} - \underline{w})(1-\tau_1)) - G(c_0)}{\lambda}. \tag{15}$$

As we did in Section 3, we can introduce the social marginal weight on earnings losses due to (marginal) involuntary unemployment: $g_0^e = [G(c_0 + (\bar{w} - \underline{w})(1-\tau_1)) - G(c_0)]/[\lambda(\bar{w} - \underline{w})(1-\tau_1)]$ in order to rewrite (15) as:

$$\frac{\bar{w} - \underline{w}}{\bar{w}} = -\frac{\tau_1}{1-\tau_1} \cdot \frac{1}{g_0^e} > 0. \tag{16}$$

We summarize all those results in the following proposition (formally proved in Appendix A.4):

Proposition 3 *Under Assumption 1 (efficient rationing), assuming $e_1 > 0$ and $\eta_1 < \infty$, if $g_1 > 1$ at the optimal tax allocation (with no minimum wage), then introducing a minimum wage is desirable. Furthermore, at the joint minimum wage and tax optimum, we have:*

- $h_0 g_0 + h_1 g_1 + h_2 g_2 = 1$ *(Social welfare weights average to one)*
- $\tau_2/(1-\tau_2) = (1-g_2)/e_2 > 0$ *(Formula for τ_2 unchanged)*
- $g_1 = 1$ *(Full redistribution to low skilled workers)*
- $(\bar{w} - \underline{w})/\bar{w} = -\tau_1/[(1-\tau_1) \cdot g_0^e] > 0$ *(Negative tax rate on low skilled work $\tau_1 < 0$)*

Quantitatively, τ_1 is primarily determined to meet the condition $g_1 = 1$. The optimal minimum wage wedge $(\bar{w} - \underline{w})/\bar{w}$ is determined by equation (16) and is increasing in the size of the absolute subsidy $|\tau_1|$ and decreasing in the social weight on unemployment earnings

[27] As usual, the changes in dw_1 and dw_2 induced by the minimum wage change do not have any fiscal consequence as we have $h_1 dw_1 + h_2 dw_2 = 0$ due to the no profit condition (5).

losses g_0^e. As discussed in Section 3, we can define the implicit market wage rate w_1 as the wage rate that would prevail under the same tax rates τ_1, τ_2, but with no minimum wage. In that case, assuming constant elasticity of supply and demand, we showed that the minimum wage markup over the market wage rate $\bar{w}/w_1$ for a given minimum wage wedge $(\bar{w}-\underline{w})/\bar{w}$ was increasing in e_1 and decreasing in η_1. This implies that our previous result (that the optimal minimum wage increases with e_1 and decreases with η_1) carries over to the case with optimal taxes. It is important to note that a high demand elasticity leads to a smaller minimum wage not because it creates more unemployment, but because a large demand elasticity makes unemployment more costly by increasing the wedge $(\bar{w}-\underline{w})/\bar{w}$.

The previous result that the optimal minimum wage follows an inverted U-shape pattern with the strength of redistributive tastes also carries over to the case with optimal taxes. Extreme redistributive (Rawlsian) tastes imply that $g_1 = 0 < 1$ and thus no minimum wage is desirable. Conversely, no redistributive tastes imply that $g_0 = g_1 = g_2 = 1$, a situation where no minimum wage is desirable.

- **A Minimum Wage with $\tau_1 > 0$ is 2nd Best Pareto Inefficient**

The last result from Proposition 3 on the negativity of τ_1 at the joint minimum wage and tax optimum has a very important corollary:

Proposition 4 *In our model with extensive labor supply responses, a binding minimum wage associated with a positive tax rate on minimum wage earnings ($\tau_1 > 0$) is second-best Pareto inefficient. This result remains a-fortiori true when rationing is not efficient.*

Proposition 4 is illustrated in Figure 4 which depicts a situation with a binding minimum wage and a positive tax rate on low skilled work $\tau_1 > 0$. Suppose that the government reduces the minimum wage ($d\bar{w} < 0$) while keeping c_0, c_1, c_2 constant. Reducing the minimum wage leads to a positive employment effect $dh_1 > 0$ as involuntary unemployment is reduced, improving the welfare of the newly employed workers and increasing tax revenue as $\tau_1 > 0$. The increase $dh_1 > 0$ also leads to a change $dw_2 > 0$. However, because $h_1 d\bar{w} + h_2 dw_2 = 0$ (through the no-profit condition (5)), the mechanical fiscal effect of $d\bar{w}$ and dw_2, keeping c_1 and c_2 constant, is zero. Because c_0, c_1, c_2 remain constant, nobody's welfare is reduced.[28] The

[28]Because, $c_2 - c_0$ remains constant, h_2 does not change either.

increase in welfare due to the reduction in unemployment remains a-fortiori true if rationing is not efficient. Therefore, this reform is a second-best Pareto improvement.

The results of Proposition 4 do not necessarily carry over to a model with general labor supply functions. For example, if workers respond along the intensive margin, the minimum wage generates not only involuntary unemployment, but also involuntary over-work as high skilled workers are also rationed out. In that case, a minimum wage decrease would induce high skilled workers to become minimum wage workers, reducing government revenue. However, the fact that the minimum wage can create over-work is rarely discussed in empirical studies, suggesting the intensive response channel is unimportant empirically.

Proposition 4 may have wide applicability because many OECD countries, especially in continental Europe, combine significant minimum wages (OECD 1998, Immervoll 2007) with very high tax rates on low skilled work (Immervoll et al. 2007). The high tax rates are generated by substantial payroll tax rates (financing social security benefits) and by the high phasing-out rates of traditional means-tested transfer programs.

In practice, the reform described in Proposition 4 could be achieved by cutting the employer payroll taxes for low income workers which lowers the (gross) minimum wage without affecting the net minimum wage after taxes and transfers.[29] Such a policy should stimulate low skill employment and increase high skill wages. Thus, the direct loss in tax revenue due to the payroll tax cut on low skilled workers could be recouped by adjusting upward taxes on high earning workers (without hurting high earning workers on net). A number of OECD countries have already implemented such policy reforms over the last 15 years.[30]

The US policy in recent decades of letting inflation erode the minimum wage while expanding the Earned Income Tax Credit is closely related. The EITC expansions compensate minimum wage workers for the erosion in the minimum wage (so that they do not lose on net) and attracts previously unemployed workers into the labor force increasing their welfare and increasing tax revenue (assuming $\tau_1 > 0$ because of the phasing-out of welfare programs). In principle, the direct fiscal cost of the EITC expansion (which maintains c_1 constant) can be recouped by increasing τ_2 as w_2 increases (so that c_2 also stays constant).

[29]Politically, it is extremely difficult to directly cut the legal minimum wage.

[30]For example, France started reducing the employer payroll tax on low income workers in the early 1990s (see Crépon and Desplatz, 2002 for an empirical analysis).

5 Numerical Simulations

• Case with no Taxes or Fixed Taxes

We make the following parametric assumptions: (1) we assume a CES production function with elasticity of substitution $\sigma > 0$; (2) we assume constant labor supply elasticities $e_i > 0$ by choosing $P_i(w) = (w/\bar{\theta}_i)^{e_i}$. Furthermore, we assume $(h_1^0, h_2^0) = (1/4, 3/4)$, and a CRRA social welfare function $G(u) = (u + B)^{1-\gamma}/(1-\gamma)$ with risk aversion parameter $\gamma > 0$ and where $B > 0$ is a constant used to avoid infinitely negative utility or infinite social marginal utility for non-workers.[31] We calibrate the production function so that $(w_1^*, w_2^*) = (1, 3)$ and the labor supply functions so that $(h_0^*, h_1^*, h_2^*) = (0.2, 0.2, 0.6)$ at the no minimum wage equilibrium. Throughout, we also assume $e_2 = 0.25$ and $B = 0.5$.

Panel A in Table 1 displays the optimal minimum wage markup over the undistorted market wage w_1^* as well as the involuntary unemployment rate (among all low skilled individuals) under various scenarios for e_1, σ, and γ. The table confirms that the optimal minimum wage is increasing in e_1 (comparing columns (1), (2), (3)), decreasing in σ (comparing columns (4), (5), (6)), and has an inverted U-shape pattern with γ (comparing panels A1, A2, and A3). The optimal minimum wage is small for a high $\gamma = 3$ value.

Panel B in Table 1 illustrates numerically that, starting from a substantial flat rate tax where $\tau_1 = \tau_2 = 0.35$ (and using the same parametrization as in Panel A), the optimal minimum wage is much lower in this case than with no taxes (and is actually useless when $\sigma = e_1 = 0.25$).

• Case with Optimal Taxes

Table 2 provides some numerical simulation illustrations using the same parametrization as in the situation with no taxes/transfers (Table 1). Table 2 shows the optimal tax rates with no minimum wage, and displays the optimal tax rates and the optimal minimum wage markup (and associated unemployment level among the unskilled) in the case of joint minimum wage/tax optimization. Table 2 confirms our key findings that the minimum wage should be associated with higher low skilled work subsidies than the case of optimal tax rates with no minimum wage. Table 2 also shows that the optimal minimum wage is increasing with e_1

[31] B could represent for example a uniform lump-sum transfer, whose cost is unaffected by behavioral responses.

and decreasing with σ. Finally, the minimum wage is useless in the high redistributive case $\gamma = 3$ as $g_1 < 1$ at the pure tax optimum.[32] Interestingly, comparing Tables 1 and 2 suggests that the minimum wage with optimal taxes is not necessarily smaller than in the case without taxes, especially when redistributive tastes are not too large ($\gamma = 0.5$).

6 Conclusion

Our paper proposes a theoretical analysis of optimal minimum wage policy for redistribution purposes in a perfectly competitive labor market, considering both the case with no taxes/ transfers and the case with optimal taxes/ transfers. In light of the previous literature on this topic, we find that the standard competitive labor market model offers a surprisingly strong case for using the minimum wage when we adopt the efficient rationing assumption. The minimum wage is a useful tool if the government values redistribution toward low wage workers, and this remains true in the presence of optimal nonlinear taxes/transfers. In that context, our model of occupational choice abstracting from hours of work allows us to overcome the informational inconsistency that plagued previous work analyzing minimum wage policy with optimal income taxation. Our model fits into the general theory of rationing developed by Guesnerie (1981) and Guesnerie and Roberts (1984) showing a minimum wage effectively rations low skilled labor. Such rationing is desirable because the optimal tax/transfer over-encourages the supply of low skilled labor.

When low skilled labor supply is along the extensive margin, as empirical studies suggest, a minimum wage should always be associated with in-work subsidies: the co-existence of minimum wages and positive participation tax rates for low skilled workers is (second-best) Pareto inefficient. In that situation (common in most OECD countries) a cut in employer payroll taxes decreasing the gross minimum wage while keeping the net minimum wage constant, combined with an offsetting tax increase on higher skilled workers is Pareto improving.

There are a number of issues that we have abstracted from in our very stylized model that are worth pointing out as caveats and potential avenues for future research.

First, as mentioned, we abstract from the hours of work decision which allows us to develop

[32] The fact that the minimum wage is zero is in large part the consequence of the two skill model assumption. A model with many skills would generate $g_1 > 1$ at the tax optimum except for extreme Rawlsian redistributive tastes. As discussed below, such a model could cast light on where in the wage distribution should the minimum wage be set.

a model with no informational inconsistencies. However, in practice, taxes and transfers are based on earnings while minimum wages are based on hourly rates. In reality, the government can observe both earnings and hours of work of employees as this information is generally included in the payroll accounting of employers and is sometimes required to be reported to the government for administering payroll taxes or maximum hours laws. Therefore, the question remains why taxes and transfers are based on earnings rather than wage rates. A possible explanation is that hours of work are not very elastic, and most of the labor supply responses take place in the form of occupation decisions, and in particular in labor force participation decisions. If hours were very elastic, taxes and transfers should be based (at least in part) on wage rates.[33] We conjecture that our results on the desirability of the minimum wage would carry over to that case as well.

Second, a minimum wage rationing mechanism operates very differently from a tax and transfer which alters prices, but lets markets clear freely. The rationing induced by the minimum wage creates an allocation problem with no natural market. It is conceivable that the allocation problem might not lead to the efficient rationing allocation (as we assumed), or that the transaction costs (search costs or queuing costs) needed to reach that efficient allocation are not negligible. Evaluating such costs using a model with frictions would be valuable.[34] It is also conceivable that rationing and the ensuing involuntary unemployment would create additional psychological costs (such as feelings of low self-worth) that are not captured in standard models (including those with search frictions), which would make minimum wage policies less attractive in practice.

Finally, our numerical simulations have been purely illustrative and it would be worth trying to calibrate these using empirically estimated parameters for the wage distribution, the elasticities of labor demand and supply, and the degree of rationing efficiency created by the minimum wage.

[33] Some transfer programs are based partly on hours information. For example, the British Working Families Credit is given only to families where one earner works at least 16 hours a week. The current US welfare program TANF imposes work requirements, which is an indirect way of conditioning transfers on hours of work.

[34] Hungerbuhler and Lehmann (2007) have made an important step in this direction by analyzing optimal minimum wage policy with optimal tax in a search model.

A Appendix: Formal Proofs

A.1 Proof of Proposition 1 and Formula (7)

Social welfare is given by:

$$SW(\bar{w}) = [1 - D_1(\bar{w}) - h_2^0 \cdot P_2(w_2)]G(0) + h_1^0 \int_0^{\underline{w}} G(\bar{w} - \theta)p_1(\theta)d\theta + h_2^0 \int_0^{w_2} G(w_2 - \theta)p_2(\theta)d\theta,$$

where $\underline{w}$ is defined as $h_1^0 \cdot P_1(\underline{w}) = D_1(\bar{w})$. We have:

$$\frac{dSW}{d\bar{w}} = -D_1'(\bar{w}) \cdot G(0) - h_2^0 \cdot \frac{dw_2}{d\bar{w}} \cdot p_2(w_2) \cdot G(0) + h_1^0 \cdot G(\bar{w} - \underline{w}) \cdot p_1(\underline{w}) \cdot \frac{d\underline{w}}{d\bar{w}} + h_1^0 \int_0^{\underline{w}} G'(\bar{w} - \theta)p_1(\theta)d\theta$$

$$+ h_2^0 \cdot \frac{dw_2}{d\bar{w}} \int_0^{w_2} G'(w_2 - \theta)p_2(\theta)d\theta + h_2^0 \cdot \frac{dw_2}{d\bar{w}} \cdot G(0) \cdot p_2(w_2).$$

The second and last term cancel out (as marginal high skill workers are indifferent between working or not). The no-profit condition $F(h_1, h_2) = w_1 h_1 + w_2 h_2$ implies that $h_1 d\bar{w} + h_2 dw_2 = 0$ so that $dw_2/d\bar{w} = -h_1/h_2$. Furthermore, $h_1^0 \cdot P_1(\underline{w}) = D_1(\bar{w})$ implies that $h_1^0 \cdot p_1(\underline{w})d\underline{w} = D_1'(\bar{w})d\bar{w}$. Therefore, we have:

$$\frac{dSW}{d\bar{w}} = D_1'(\bar{w})[G(\bar{w} - \underline{w}) - G(0)] + h_1^0 \int_0^{\underline{w}} G'(\bar{w} - \theta)p_1(\theta)d\theta - h_1^0 \cdot \frac{P_1}{P_2} \int_0^{w_2} G'(w_2 - \theta)p_2(\theta)d\theta$$

$$= -\eta_1 \cdot g_0^e \cdot \frac{\bar{w} - \underline{w}}{\bar{w}} \cdot h_1 \cdot \lambda + [g_1 - g_2] \cdot h_1 \cdot \lambda,$$

where we used the definitions of η_1, g_0^e, g_1, g_2 in the last equality. Thus, starting from the competitive equilibrium where $\bar{w} = \underline{w} = w_1^*$, the first term is zero, making the minimum wage desirable if and only if $g_1 > g_2$, hence proving Proposition 1. At the optimum $\bar{w}$, $dSW/d\bar{w} = 0$ which leads immediately to formula (7). $\square$

A.2 Proof of Proposition 2

Social welfare is given by:

$$SW(\bar{w}) = [1 - D_1(\bar{w}) - h_2^0 \cdot P_2(w_2(1 - \tau_2))]G(c_0) + h_1^0 \int_0^{\underline{w}(1-\tau_1)} G(c_0 + \bar{w}(1 - \tau_1) - \theta)p_1(\theta)d\theta$$

$$+ h_2^0 \int_0^{w_2(1-\tau_2)} G(c_0 + w_2(1 - \tau_2) - \theta)p_2(\theta)d\theta,$$

where $\underline{w}$ is defined as $h_1^0 \cdot P_1(\underline{w}(1 - \tau_1)) = D_1(\bar{w})$. The government budget constraint is $c_0 \leq D_1(\bar{w})\tau_1 \bar{w} + h_2^0 P_2(w_2(1 - \tau_2))\tau_2 w_2$. We denote by λ the multiplier of the budget constraint and we introduce the Lagrangian

$$L = SW(\bar{w}) + \lambda \cdot [D_1(\bar{w})\tau_1 \bar{w} + h_2^0 P_2(w_2(1 - \tau_2))\tau_2 w_2 - c_0].$$

The first order condition with respect to c_0 is:

$$\frac{dL}{dc_0} = h_0 G'(c_0) + h_1^0 \int_0^{\underline{w}(1-\tau_1)} G'(c_1-\theta)p_1(\theta)d\theta + \int_0^{w_2(1-\tau_2)} G'(c_2-\theta)p_2(\theta)d\theta - \lambda = 0.$$

Using the definitions of g_0, g_1, g_2, we obtain immediately $h_1 g_0 + h_1 g_1 + h_2 g_2 = 1$.

Starting from the competitive equilibrium with no minimum wage $\bar{w} = \underline{w} = w_1$, we have:

$$\frac{dL}{d\bar{w}}|_{\bar{w}=w_1} = -D_1'(w_1) \cdot G(c_0) - h_2^0(1-\tau_2) \cdot \frac{dw_2}{d\bar{w}} \cdot p_2(w_2(1-\tau_2)) \cdot G(c_0) +$$

$$h_1^0 \cdot G(c_0) \cdot p_1(w_1(1-\tau_1)) \cdot (1-\tau_1) \frac{d\underline{w}}{d\bar{w}}|_{\bar{w}=w_1} + h_1^0(1-\tau_1) \int_0^{w_1(1-\tau_1)} G'(c_0 + (1-\tau_1)w_1 - \theta)p_1(\theta)d\theta$$

$$+ h_2^0 \cdot (1-\tau_2)\frac{dw_2}{d\bar{w}} \int_0^{w_2(1-\tau_2)} G'(c_0 + w_2(1-\tau_2) - \theta)p_2(\theta)d\theta + h_2^0 \cdot (1-\tau_2)\frac{dw_2}{d\bar{w}} \cdot G(c_0) \cdot p_2(w_2(1-\tau_2))$$

$$+\lambda \cdot \left[D_1(w_1)\tau_1 + D_1'(w_1)\tau_1 w_1 + \tau_2 h_2 \frac{dw_2}{d\bar{w}} + w_2(1-\tau_2)h_2^0 p_2(w_2(1-\tau_2))\tau_2 \frac{dw_2}{d\bar{w}} \right].$$

The second and sixth terms cancel out. From $h_1^0 \cdot P_1(\underline{w}(1-\tau_1)) = D_1(\bar{w})$, we have $h_1^0 \cdot p_1(w_1(1-\tau_1))(1-\tau_1)d\underline{w}/d\bar{w} = D_1'(w_1)$ at $\bar{w} = \underline{w} = w_1$. The first and third terms cancel out. The no-profit condition $F(h_1, h_2) = \bar{w}h_1 + w_2 h_2$ implies $h_1 d\bar{w} + h_2 dw_2 = 0$ and hence $dw_2/d\bar{w} = -h_1/h_2$. Thus, using the definitions $e_2 = w_2(1-\tau_2) \cdot p_2/P_2$ and $\eta_1 = -w_1 D_1'/h_1$, we have:

$$\frac{dL}{d\bar{w}}|_{\bar{w}=w_1} = (1-\tau_1)h_1 g_1 \cdot \lambda + (1-\tau_2)h_2 g_2 \cdot (-h_1/h_2) \cdot \lambda + \lambda \cdot [h_1\tau_1 - \eta_1 h_1 \tau_1 + h_2(1+e_2)\tau_2 \cdot (-h_1/h_2)].$$

Hence,

$$\frac{1}{\lambda \cdot h_1} \cdot \frac{dL}{d\bar{w}}|_{\bar{w}=w_1} = (1-\tau_1) \cdot g_1 - (1-\tau_2) \cdot g_2 + \tau_1 - \eta_1 \cdot \tau_1 - \tau_2 \cdot (1+e_2),$$

which is condition (11) in Proposition 2. $\square$

A.3 Optimal Tax Formulas (13) with no Minimum Wage

Let us introduce $\Delta c_1 = c_1 - c_0$ and $\Delta c_2 = c_2 - c_0$. The government chooses $c_0, \Delta c_1, \Delta c_2$ to maximize social welfare SW subject to its budget constraint $h_0 c_0 + h_1 c_1 + h_2 c_2 \leq w_1 h_1 + w_2 h_2$, which can be rewritten as $c_0 + h_1 \Delta c_1 + h_2 \Delta c_2 \leq h_1 w_1 + h_2 w_2$. Therefore, the Lagrangian of the government maximization problem can be written as:

$$L = (1 - h_1^0 \cdot P_1(\Delta c_1) - h_2^0 \cdot P_2(\Delta c_2))G(c_0) + h_1^0 \int_0^{\Delta c_1} G(c_0 + \Delta c_1 - \theta)p_1(\theta)d\theta + h_2^0 \int_0^{\Delta c_2} G(c_0 + \Delta c_2 - \theta)p_2(\theta)d\theta$$

$$+\lambda \cdot [h_1^0 P_1(\Delta c_1)(w_1 - \Delta c_1) + h_2^0 P_2(\Delta c_2)(w_2 - \Delta c_2) - c_0],$$

The first order condition in c_0 (keeping Δc_1 and Δc_1 constant) is:

$$\frac{dL}{dc_0} = h_0 G'(c_0) + h_1^0 \int_0^{\Delta c_1} G'(c_1 - \theta) p_1(\theta) d\theta + \int_0^{\Delta c_2} G'(c_2 - \theta) p_2(\theta) d\theta - \lambda = 0.$$

Using the definitions of g_0, g_1, g_2, we obtain immediately $h_1 g_0 + h_1 g_1 + h_2 g_2 = 1$. The first order condition in Δc_1 is:

$$0 = \frac{dL}{d\Delta c_1} = -h_1^0 \cdot p_1(\Delta c_1) G(c_0) + h_1^0 \int_0^{\Delta c_1} G'(c_1 - \theta) p_1(\theta) d\theta + h_1^0 G(c_0) p_1(\Delta c_1)$$

$$+\lambda \left[h_1^0 p_1(\Delta c_1)(w_1 - \Delta c_1) - h_1^0 P_1(\Delta c_1) + h_1 \cdot \frac{dw_1}{d\Delta c_1} + h_2 \cdot \frac{dw_2}{d\Delta c_1} \right].$$

The first and third term cancel out (with no minimum wage, marginal low skilled workers are indifferent between working or not working). The no-profit condition $F(h_1, h_2) = w_1 h_1 + w_2 h_2$ implies that $h_1 dw_1 + h_2 dw_2 = 0$ and hence $h_1 dw_1 / d\Delta c_1 + h_2 dw_2 / d\Delta c_1 = 0$ so that the last two terms cancel out. Therefore, we have:

$$0 = \frac{1}{\lambda} \frac{dL}{d\Delta c_1} = h_1 \cdot g_1 - h_1 + h_1 \frac{w_1 - \Delta c_1}{\Delta c_1} \cdot \frac{\Delta c_1 \cdot p_1(\Delta c_1)}{P_1(\Delta c_1)}.$$

Recognizing that $\Delta c_1 = w_1(1 - \tau_1)$, we have $w_1 - \Delta c_1 = w_1 \tau_1$, and by definition $e_1 = \Delta c_1 \cdot p_1 / P_1$, therefore:

$$0 = \frac{1}{h_1 \cdot \lambda} \frac{dL}{d\Delta c_1} = g_1 - 1 + \frac{\tau_1}{1 - \tau_1} \cdot e_1,$$

which implies equation (13) for $i = 1$. The proof for $i = 2$ is exactly the same. □

A.4 Proof of Proposition 3

The government chooses $c_0, \Delta c_1, \Delta c_2, \bar{w}$ to maximize social welfare SW subject to its budget constraint $c_0 + h_1 \Delta c_1 + h_2 \Delta c_2 \leq h_1 w_1 + h_2 w_2$. The Lagrangian of the government maximization problem is:

$$L = (1 - D_1(\bar{w}) - h_2^0 P_2(\Delta c_2)) G(c_0) + h_1^0 \int_0^{\bar{\theta}} G(c_0 + \Delta c_1 - \theta) p_1(\theta) d\theta + h_2^0 \int_0^{\Delta c_2} G(c_0 + \Delta c_2 - \theta) p_2(\theta) d\theta$$

$$+\lambda \cdot [D_1(\bar{w})(\bar{w} - \Delta c_1) + h_2^0 P_2(\Delta c_2)(w_2 - \Delta c_2) - c_0].$$

where $\bar{\theta}$ in the first integral term is defined so that the number of low skilled workers exactly meets the demand: $h_1^0 \cdot P_1(\bar{\theta}) = D_1(\bar{w})$. The first order condition with respect to c_0 (keeping

Δc_1 and Δc_2, and $\bar{w}$ constant) implies $h_1 g_0 + h_1 g_1 + h_2 g_2 = 1$ (same proof as in Appendix A.3, note that $\bar{w}$ constant implies w_2 is constant through the no-profit condition). Similarly, the first order condition with respect to Δc_2 implies $\tau_2/(1-\tau_2) = (1-g_2)/e_2$.

The first order condition with respect to Δc_1 is ($\bar{w}$ constant implies w_2 is constant through the no-profit condition):

$$0 = \frac{dL}{d\Delta c_1} = h_1^0 \int_0^{\bar{\theta}} G'(c_1 - \theta) p_1(\theta) d\theta - \lambda \cdot D_1(\bar{w}),$$

which implies $g_1 = 1$.

Finally, the first order condition with respect to $\bar{w}$ is:

$$0 = \frac{dL}{d\bar{w}} = -D_1'(\bar{w}) G(c_0) + h_1^0 \cdot \frac{d\bar{\theta}}{d\bar{w}} G(c_0 + \Delta c_1 - \bar{\theta}) p_1(\bar{\theta}) + \lambda \cdot \left[D_1'(\bar{w})(\bar{w} - \Delta c_1) + D_1(\bar{w}) + h_2 \frac{dw_2}{d\bar{w}} \right].$$

By definition of τ_1, we have $\Delta c_1 = \bar{w}(1-\tau_1)$. Introducing the reservation wage $\underline{w}$ of the marginal worker defined as $\underline{w}(1-\tau_1) = \bar{\theta}$ as in the text, and noting that $h_1^0 \cdot P_1(\bar{\theta}) = D_1(\bar{w})$, we have $h_1^0 \cdot p_1(\bar{\theta}) d\bar{\theta}/d\bar{w} = D_1'(\bar{w})$. Finally, the no-profit condition $F(h_1, h_2) = \bar{w} h_1 + w_2 h_2$ implies $h_1 d\bar{w} + h_2 dw_2 = 0$ and hence $dw_2/d\bar{w} = -h_1/h_2$. As a result, the last two terms in the squared expression cancel out. Hence, we have:

$$0 = \frac{dL}{d\bar{w}} = D_1'(\bar{w})[G(c_0 + (1-\tau_1)(\bar{w} - \underline{w})) - G(c_0)] + \lambda \cdot D_1'(\bar{w}) \bar{w} \tau_1,$$

which implies

$$-\frac{\tau_1}{1-\tau_1} = \frac{\bar{w} - \underline{w}}{\bar{w}} \cdot \frac{G(c_0 + (\bar{w} - \underline{w})(1-\tau_1)) - G(c_0)}{\lambda(\bar{w} - \underline{w})(1-\tau_1)} = \frac{\bar{w} - \underline{w}}{\bar{w}} \cdot g_0^e,$$

where we have used the definition g_0^e in the last equality. $\square$

B Appendix: Extensions

B.1 Uniform Rationing

As discussed, our previous results are derived under the key assumption of efficient rationing, the situation most favorable to the minimum wage. Below we briefly explore how results change if we adopt the polar opposite "uniform rationing" assumption whereby unemployment is distributed across workers independently of surplus.[35]

• **Case with no Taxes**

In the case of uniform rationing with no taxes, the government chooses $\bar{w}$ to maximize:

$$SW = (1 - D_1(\bar{w}) - h_2^0 P_2(w_2))G(0) + D_1(\bar{w}) \int_0^{\bar{w}} G(\bar{w} - \theta) \frac{p_1(\theta)}{P_1(\bar{w})} d\theta + h_2^0 \int_0^{w_2} G(w_2 - \theta) p_2(\theta) d\theta. \tag{17}$$

The second term in equation (17) reflects the notion that all workers with work costs $\theta \in (0, \bar{w})$ have the same probability of being employed, but that the total number of low skilled workers is given by the demand function $D_1(\bar{w})$.

Suppose that $\bar{w}$ is increased by $d\bar{w}$ under the "uniform rationing" scenario. The redistributive value of introducing a small minimum wage $d\bar{w}$ remains the same: $T = [g_1 - g_2]h_1 d\bar{w}$. The minimum wage reduces employment through a demand effect by $dh_1 = -\eta_1 h_1 d\bar{w}/\bar{w}$. However, the minimum wage will induce workers with cost of work $\theta \in (\bar{w}, \bar{w} + d\bar{w})$ to look for a job as well. There are $e_1 h_1^S d\bar{w}/\bar{w}$ such workers where $h_1^S = h_0^1 P_1(\bar{w})$ is the number of low skilled individuals willing to work for wage $\bar{w}$. Under efficient rationing, those marginal workers would stay out of work. Under uniform rationing, however, a fraction h_1/h_1^S of those new workers will join the labor force and will displace other workers as unemployment is distributed uniformly. That excess labor supply creates involuntary unemployment. As involuntary unemployment is distributed uniformly across all low skilled workers, the average welfare cost per displaced worker is $\int_0^{\bar{w}} [G(\bar{w} - \theta) - G(0)] p_1(\theta) d\theta / P_1(\bar{w})$. The number of displaced workers is $h_1(e_1 + \eta_1) d\bar{w}/\bar{w}$. Thus, the welfare loss due to involuntary unemployment is equal to $U = -h_1(d\bar{w}/\bar{w})(e_1 + \eta_1) \int_0^{\bar{w}} [G(\bar{w} - \theta) - G(0)] p_1(\theta) d\theta / P_1(\bar{w})$. At the optimum, we

[35] "Uniform rationing" amounts to assuming that unemployment strikes randomly and that Coasian re-trading is prohibitively expensive and hence does not happen at all.

have $U + T = 0$ which implies

$$\int_0^{\bar{w}} \frac{[G(\bar{w}-\theta)-G(0)]p_1(\theta)d\theta}{\bar{w}P_1(\bar{w})} \cdot (e_1+\eta_1) = g_1 - g_2. \tag{18}$$

If at $\bar{w} = w_1^*$, the left-hand-side is smaller than the right-hand-side of (18), then a minimum wage is desirable (and conversely for the alternative case). The key point is that a minimum wage is not necessarily desirable under "uniform rationing."

We can introduce a welfare weight on employment losses defined as $g_0^u = \int_0^{\bar{w}}[G(\bar{w}-\theta) - G(0)]p_1(\theta)d\theta / \int_0^{\bar{w}}(\bar{w}-\theta)p_1(\theta)d\theta$. If we assume that the supply elasticity e_1 is constant, then $P_1(\theta) = C \cdot \theta^{e_1}$ and hence $\int_0^{\bar{w}}(\bar{w}-\theta)p_1(\theta)d\theta / P_1(\bar{w}) = \bar{w}/(1+e_1)$. In this case, we can rewrite (18) as follows:

$$\frac{e_1+\eta_1}{1+e_1} = \frac{g_1-g_2}{g_0^u}. \tag{19}$$

This equation is an implicit formula for the optimal minimum wage. Presumably, the welfare weight ratio $(g_1-g_2)/g_0^u$ is decreasing with $\bar{w}$. Formula (19) implies that the minimum wage should be increased up to the point where the welfare weight ratio is equal to the elasticity ratio $(e_1+\eta_1)/(1+e_1)$. Obviously, if at $\bar{w} = w_1^*$, the welfare weight ratio is already below the elasticity ratio, then no minimum wage is desirable. Note that the elasticity ratio is increasing in η_1 and, hence, the optimum minimum wage is decreasing in η_1. If $g_0^u \geq g_1$,[36] equation (19) implies that the right-hand-side is less than one, and a minimum wage will only be desirable if $\eta_1 < 1$.

When $\eta_1 < 1$, the elasticity ratio increases with e_1. This implies that the optimum minimum wage is decreasing in e_1. This contrasts with our results under efficient rationing and can be understood as follows: a large supply elasticity makes unemployment less costly as workers have lower surplus from working on average, but a large supply elasticity induces more formerly out of work individuals to look for jobs, displacing workers with higher surpluses (which is inefficient). When $\eta_1 < 1$, the latter effect is stronger than the former effect explaining why the minimum wage decreases with e_1.[37]

Empirically, we would expect rationing in practice to be in-between efficient rationing and uniform rationing. It is very easy to extend the model to a mixed situation where a fraction

[36] For example, this holds for constant supply elasticity e_1 and constant risk aversion functions $G(.)$.

[37] Note also that, with uniform rationing, and if workers can smooth consumption across unemployment spells, then we have $g_0^u = g_1$. The standard result about the pivotal $\eta_1 = 1$ can be seen as a particular case of (19) when $e_1 = 0$ (no supply elasticity), $g_2 = 0$ (no value assigned to high skilled workers), and $g_0^u = g_1$ (unemployment spells are shared and consumption is smoothed).

δ of unemployment strikes uniformly while a fraction $1-\delta$ of the unemployment is efficiently allocated. In that case, the formula for the optimum minimum wage is a straight average of (7) and (19), namely $g_1 - g_2 = (1-\delta) g_0^e \eta_1 (\bar{w} - \underline{w})/\bar{w} + \delta g_0^u (\eta_1 + e_1)/(\eta_1 + e_1)$. This shows that our efficient rationing results are *robust* to the introduction of a little bit of uniform rationing.

• Case with Optimal Taxes

In the case with taxes, the government chooses, c_0, c_1, c_2, and $\bar{w}$ to maximize:

$$SW = (1 - D_1(\bar{w}) - h_2^0 P_2(c_2 - c_0)) G(c_0) + D_1(\bar{w}) \int_0^{c_1 - c_0} \frac{G(c_1 - \theta) p_1(\theta)}{P_1(c_1 - c_0)} d\theta + h_2^0 \int_0^{c_2 - c_0} G(c_2 - \theta) p_2(\theta) d\theta, \tag{20}$$

subject to the standard budget constraint and the fact that demand for labor is competitively set. The second term in Equation (20) reflects the notion that all workers with work costs $\theta \in (0, c_1 - c_0)$ have the same probability of being employed, but that the total number of low skilled workers is given by the demand function $D_1(\bar{w})$. The first order condition with respect to c_0 (keeping $c_1 - c_0, c_2 - c_0, \bar{w}$ constant) implies the standard result $h_0 g_0 + h_1 g_1 + h_2 g_2 = 1$. The first order condition with respect to c_2 leads to the standard optimal tax formula for τ_2, namely $\tau_2/(1-\tau_2) = (1-g_2)/e_2$. The first order condition with respect to c_1 leads to:

$$\frac{g_1 - 1}{e_1} = g_0^u \cdot \int_0^{c_1 - c_0} \left(1 - \frac{\theta}{c_1 - c_0}\right) \frac{p_1(\theta)}{P_1(c_1 - c_0)} d\theta, \tag{21}$$

where $g_0^u = \int_0^{c_1 - c_0} [G(c_1 - \theta) - G(c_0)] p_1(\theta) d\theta / (\lambda \cdot \int_0^{c_1 - c_0} (c_1 - c_0 - \theta) p_1(\theta) d\theta)$ is the welfare weight on (marginal) unemployment losses.

The first order condition with respect to $\bar{w}$ leads to:

$$-\frac{\tau_1}{1 - \tau_1} = g_0^u \cdot \int_0^{c_1 - c_0} \left(1 - \frac{\theta}{c_1 - c_0}\right) \frac{p_1(\theta)}{P_1(c_1 - c_0)} d\theta. \tag{22}$$

Therefore and strikingly, combining those two first order conditions, we find that the optimal tax formula for τ_1 *in the presence of the optimal minimum wage* is the same as with no minimum wage, namely $\tau_1/(1-\tau_1) = (1-g_1)/e_1$. Intuitively and following the derivation from Figure 2b, this can be understood as follows: suppose c_1 is increased by dc_1, and at the same time the minimum wage $\bar{w}$ is reduced by $d\bar{w}$ such that $dc_1 \cdot p_1/P_1 = d\bar{w} \cdot D_1'(\bar{w})/D_1$. In that case, a fraction D_1/P_1 of those $p_1 dc_1$ workers willing to join the labor force because of dc_1 can do so and hence the fiscal effect of the reform is $(T_1 - T_0) p_1 dc_1 \cdot D_1/P_1 = D_1 dc_1 \cdot e_1 \cdot \tau_1/(1-\tau_1)$ and the standard formula goes through. We can then obtain the following proposition:

Proposition 5 *With optimal taxes/transfers and uniform rationing, if the welfare weight on unemployment losses is larger than the welfare weight on low skilled workers ($g_0^u \geq g_1$) and the supply elasticity e_1 is constant, then a minimum wage is not desirable.*

Proof: Under the assumption of a constant e_1 and if a binding minimum wage, the integral term in the right-hand-side of (21) is equal to $1/(1+e_1)$ and hence (21) can be rewritten as $(g_1 - 1)/e_1 = g_0^u/(1+e_1)$. However, $(g_1 - 1)/e_1 < g_1/(1+e_1) \leq g_0^u/(1+e_1)$, where the first inequality follows from that fact that $g_1 < 1 + e_1$ (as $\tau_1 = (1 - g_1)/(1 - g_1 + e_1)$)[38] and the second inequality from our assumption that $g_1 \leq g_0^u$. This creates a contradiction showing that the minimum wage cannot be binding. $\square$

As in the case with no taxes, it is easy to extend the model to a mixed situation where a fraction δ of unemployment strikes uniformly, while a fraction $1 - \delta$ of the unemployment is efficiently allocated. In particular, a minimum is desirable if and only if $g_1 > 1+\delta g_0^u e_1/(1+e_1)$ at the tax optimum with no minimum wage. When a minimum wage is desirable, at the optimum we have, $g_1 = 1+\delta g_0^u e_1/(1+e_1)$ and $-\tau_1/(1-\tau_1) = \delta g_0^u/(1+e_1)+(1-\delta)g_0^e(\bar{w}-\underline{w})/\bar{w}$. This shows that our results under efficient rationing are also robust to the introduction of a little bit of uniform rationing (small δ) in the case with optimal taxes.

B.2 General Labor Supply Function

We consider a general model with I occupations (instead of 2) and a general production function.[39] Most importantly, the model allows for any labor supply responses, instead of only considering the extensive margin (as discussed previously).

- **Model and Optimal Taxation**

The model we use is the general occupation model described in the appendix of Saez (2002) and Saez (2004). There are $I + 1$ occupations, paying wages $w_0 = 0, w_1, .., w_I$. Occupation 0 denotes unemployment. There is a constant return to scale production function $F(h_1, .., h_I)$ so that $w_i = \partial F/\partial h_i$. We assume that in equilibrium, occupations are ordered so that $0 < w_1 < .. < w_I$. Each individual is characterized by a cost parameter $\theta = (\theta_0 = 0, \theta_1, .., \theta_I)$,

[38]If $g_1 > 1 + e_1$, then reducing τ_1 is strictly desirable which cannot happen at the optimum.

[39]Introducing a capital input would also be possible as long as we assume that returns on capital can be taxed at a specific rate τ_K. Similarly, pure profits can also be introduced as long as the government can tax them away fully.

which describes the labor supply cost for the individual to work in each occupation $i = 1, .., I$. By assumption, being out of work is costless. We assume that θ is distributed according to a measure $\nu(\theta)$ on Θ, with total population normalized to one.

The government can apply a general income tax and transfer system $T = (T_0, .., T_I)$. We denote by $c_i = w_i - T_i$ the disposable income (after taxes/transfers) in occupation i. An individual with cost θ picks the occupation i which maximizes $c_j - \theta_j$ for $j = 0, .., I$. Hence, the set Θ is partitioned into $I + 1$ subsets $\Theta_0, .., \Theta_I$ so that individuals with $\theta \in \Theta_i$ choose occupation i. We denote by $h_i = \nu(\Theta_i)$ the fraction of individuals in occupation i. The supply functions are functions of $c = (c_0, .., c_I)$ and are denoted by $h_i(c_0, .., c_I)$. We assume that θ is distributed smoothly across individuals so that the supply functions h_i are continuously differentiable. This is a fully general supply model with no income effects. The participation model from our previous section is a special case of this model. Similarly, the intensive labor supply of Mirrlees (1971) can be represented in this discrete model by assuming that individuals of "type i" can work in job $i - 1$ at no cost or work in job i at cost $\theta_i > 0$ (see Saez, 2002 for details).

Abstracting first from the minimum wage, the government chooses $c = (c_0, .., c_I)$ in order to maximize: $SW = \int_{\theta \in \Theta} G(c_i - \theta_i) d\nu(\theta)$ subject to the budget constraint: $\sum_{j=0}^{I} (w_j - c_j) \cdot h_j(c) \geq 0$. $G(.)$ is increasing and concave, and where index i inside in integral for SW denotes the utility maximizing job choice of individual θ. We denote again by λ the multiplier of the budget constraint.

The first order condition with respect to c_i is simply:

$$(1 - g_i) h_i = \sum_{j=0}^{I} T_j \cdot \frac{\partial h_j}{\partial c_i}, \tag{23}$$

where g_i is the average social marginal welfare weight in occupation i, defined as $g_i = \int_{\theta \in \Theta_i} G'(c_i - \theta_i) d\nu(\theta) / (\lambda \cdot h_i)$.

The derivation is straightforward once one recognizes: (1) the welfare effect of a small increase dc_i due to switching jobs or behavioral responses is zero (because of a standard envelope theorem argument) and (2) the wage changes $dw_1, .., dw_I$ due to dc_i have no fiscal consequence due to the no-profit condition $F = w_1 h_1 + .. + w_I h_I$, which implies that $h_1 dw_1 + .. + h_I dw_I = 0$.

The no income effects assumption implies $\sum_{j=0}^{I} g_i \cdot h_i = 1$. This can be obtained by increasing every c_i by dc uniformly. This generates no behavioral responses and hence the fiscal cost dc must be equal to the welfare gain $dc \cdot \sum_j h_j g_j$. This implies that the average of g_i is one.

• **Desirability of Minimum Wage Rationing**

We can generalize Proposition 3 as follows: under efficient rationing, if $g_1 > 1$ at the tax optimum, introducing a minimum wage is desirable.

The proof remains the same: starting from the tax optimum with no minimum wage, setting $\bar{w} = w_1$ and increasing c_1 improves social welfare when $g_1 > 1$ without triggering any behavioral response because those who would like to move to occupation 1 cannot do so because of the minimum wage rationing. The efficient rationing assumption also means those already in occupation 1 are not displaced.

Theoretically, the occupation model can be seen as a generalized Diamond-Mirrlees optimal tax model, which inherits most of the structure and properties of that model. In particular, the analysis of minimum wages parallels the theory of rationing in second-best optimal tax models developed by Guesnerie (1981) and Guesnerie and Roberts (1984). Following Samuelson (1951), using the symmetry result ($\partial h_i/\partial c_j = \partial h_j/\partial c_i$), the optimal tax formula (23) can be rewritten as:

$$\frac{1}{h_i}\sum_{j=0}^{I} -T_j \cdot \frac{\partial h_i}{\partial c_j} = g_i - 1. \tag{24}$$

The left-hand-side measures the percentage change in h_i created by the tax system (which changes c_j from w_j to $w_j - T_j$). Hence, if $g_i > 1$ ($g_i < 1$), the optimal tax system encourages (discourages) the supply of labor in occupation i. Therefore, the optimal tax system (absent a minimum wage) subsidizes goods going to disadvantaged individuals (here low skilled work). As a result, low skilled work is socially over-supplied at the second best tax optimum. It is then socially desirable to ration subsidized low skill labor using a minimum wage.[40]

The "full redistribution to minimum wage workers" result of Proposition (3) also extends to this general model. At the joint tax and minimum wage optimum, the optimum minimum wage $\bar{w}$ covers occupations $i = 1, .., i^*$ (we assumed occupations were ordered). Thus those

[40]In the (discrete) intensive labor supply, the tax rate between occupation 0 (no work) and occupation 1 (lowest paid occupation) is positive (Saez, 2002) as in the Mirrlees continuous model. Nevertheless, it is still the case that low skilled work is over-encouraged by the tax system because there are more individuals who shift from occupation 2 to occupation 1 because of taxes than individuals who shift from occupation 1 to occupation 0.

occupations pay the same wage $\bar{w}$. As a result, the government can no longer distinguish across occupations and is forced to tax (or subsidize) them uniformly, making $c_1 = .. = c_{i^*} = \bar{c}$. We denote by $\bar{T} = \bar{w} - \bar{c}$ the net tax on minimum wage workers.

Again, increasing $\bar{c}$ does not produce any behavioral labor supply response (as occupations $1, .., i^*$ are rationed by the minimum wage). Hence, the government should increase $\bar{c}$ up to the point that $\bar{g} = 1$ where $\bar{g} = (h_1 g_1 + .. + h_{i^*} g_{i^*})/(h_1 + .. + h_{i^*})$ is the average social marginal welfare weight on minimum wage workers.

• **Many Consumption Goods and Production Efficiency**

It is also possible to extend the tax model to a situation with many goods. In that context, we can show that the standard theorems of public finance (namely, the production efficiency theorem of Diamond and Mirrlees (1971) and the no commodity taxation result of Atkinson and Stiglitz (1976)) carry over to the model with optimal minimum wage with taxes/transfers.

The production efficiency theorem implies that at the joint minimum wage and tax optimum there should be production efficiency: producers should maximize profits using pre-tax prices for labor inputs and consumption outputs. This result is trivial to verify in the two skill model and remains true with many labor inputs and many consumption goods. As is well known, the production efficiency result implies that there should be no tariffs in the context of an open economy. This important result also applies when the government uses a minimum wage optimally.

Atkinson and Stiglitz (1976) suggests that, if utility functions are separable between consumption goods and labor costs and the sub-utility of consumption is homogenous across all consumers, then the optimum tax/minimum wage system should tax labor only and not impose any differentiated taxes on consumption goods. This result also carries over to the joint tax and minimum wage optimum.

B.3 Income Effects

In order to introduce income effects in the two-skill model used in the text, we can define individual utility as $u(c) - \theta \cdot l$ where $u(.)$ is increasing and concave. Thus, an individual of skill i works if and only if $\theta \leq u(c_i) - u(c_0)$. Denoting $u_i = u(c_i)$, for $i = 0, 1, 2$, the labor supply function becomes $h_i = h_i^0 \cdot P_i(u_i - u_0)$. We can again evaluate social welfare as:

$$SW = (1 - h_1 - h_2)G(u_0) + h_1^0 \int G(u_1 - \theta)p_1(\theta)d\theta + h_2^0 \int_0^{u_2 - u_1} G(u_2 - \theta)p_2(\theta)d\theta, \quad (25)$$

where $G(.)$ is a concave and increasing transformation. Note that (25) is identical to social welfare with no income effects once c_i substituted by u_i.

Let us denote again by λ the multiplier of the government budget constraint $h_0c_0 + h_1c_1 + h_2c_2 \leq h_1w_1 + h_2w_2$ which can be rewritten as:

$$h_0u^{-1}(u_0) + h_1u^{-1}(u_1) + h_2u^{-1}(u_2) \leq h_1w_1 + h_2w_2.$$

We define social marginal welfare weights as: $g_0 = G'(u_0)u'(c_0)/\lambda$ and $g_i = \int G'(u_i - \theta)p_i(\theta)d\theta \cdot u'(c_i)/\lambda$ for $i = 1, 2$.

With no minimum wage, the government chooses c_0, c_1, c_2 (or equivalently u_0, u_1, u_2) to maximize SW subject to its budget constraint. Increasing u_0, u_1, and u_2 by du leads to the first order condition:

$$h_0G'(u_0) + h_1^0 \int G'(u_1 - \theta)p_1 d\theta + h_1^0 \int G'(u_1 - \theta)p_1 d\theta = \lambda \cdot \sum_i \frac{h_i}{u'(c_i)},$$

which can be rewritten as:

$$\tilde{h}_0g_0 + \tilde{h}_1g_1 + \tilde{h}_2g_2 = 1,$$

where $\tilde{h}_i = (h_i/u'(c_i))/(\sum_j h_j/u'(c_j)) > 0$ can be interpreted as occupation shares re-normalized by the marginal utility of consumption. Using those weights, the social marginal weights again g_i average to one.

The first order condition with respect to u_i leads to the usual optimal tax formula $\tau_i/(1 - \tau_i) = (1 - g_i)/e_i$ where the supply elasticity is defined as $e_i = [(c_i - c_0)/h_i]\partial h_i/\partial c_i|_{c_0} = (c_i - c_0)u'(c_i) \cdot p_i(u_i - u_0)/P_i(u_i - u_0)$.

Again, we can show a minimum wage is desirable if $g_1 > 1$ at the tax optimum. At the joint minimum wage and tax optimum, we have $\tilde{h}_0g_0 + \tilde{h}_1g_1 + \tilde{h}_2g_2 = 1$, $g_1 = 1$, $\tau_2/(1 - \tau_2) = (1 - g_2)/e_2$. Furthermore, the first order condition in $\bar{w}$ takes a similar form $[G(u_1 - \bar{\theta}) - G(u_0)]/\lambda = -w_1 \cdot \tau_1 < 0$ (where $\bar{\theta}$ is the cost of work of the marginal worker). Hence, Proposition 4 showing that $\tau_1 > 0$ along with a binding minimum wage is Pareto dominated applies to the case with income effects as well.

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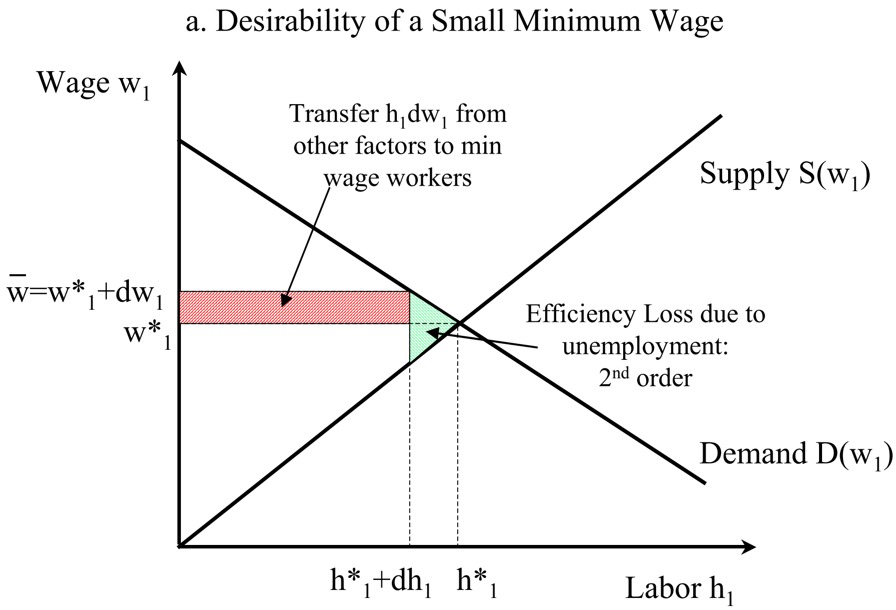

Figure 1. Minimum Wage with no Taxes and Transfers

Panel a displays the desirability of introducing a small minimum wage starting from the competitive equilibrium. A small minimum wage creates a first order transfer to low skilled workers from other factors and a second order welfare low due to involuntary unemployment (under the key assumption of uniform rationing).

Panel b displays the trade-off for setting the optimal minimum wage. Increasing the minimum wage slightly generates a first order transfer to low skilled workers from other factors and a first order loss due to involuntary unemployment. At the optimum, those two effects should be of equal size.

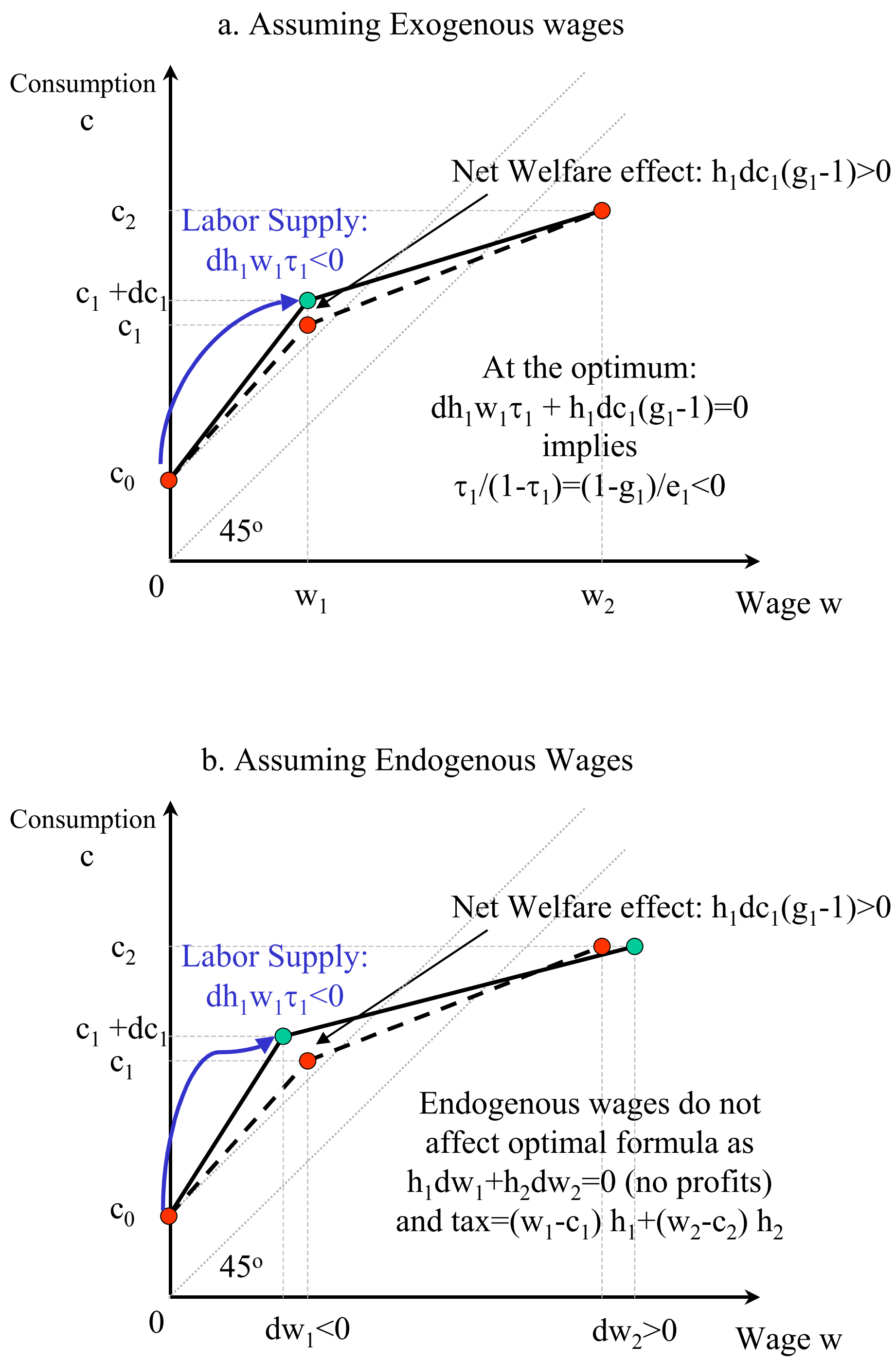

Figure 2. Optimal Income Tax Derivation (with no minimum wage)

Panel a displays the trade-offs involved when increasing c1 by dc_1 and assuming that wage rates remain fixed. At the optimum, the net welfare effect of dc_1 must equal the fiscal loss due to the behavioral response. We assume that $g_1>1$ so that the net welfare effect is positive.

Panel b shows that the derivation remains valid with endogenous wages as the fiscal effects due to changes in wages cancel out because of the no-profit condition.

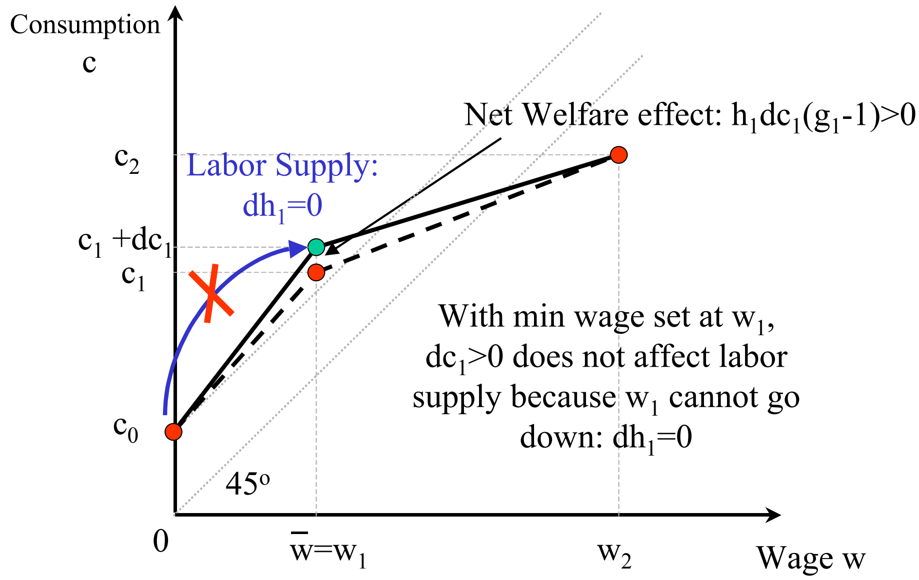

Figure 3. Desirability of a Minimum Wage under Optimal Taxes

The Figure shows that, starting from the tax optimum with no taxes (derived on Figure 2), introducing a minimum wage (equal to w_1) and increasing c_1 by dc_1 improves welfare when $g_1>1$.

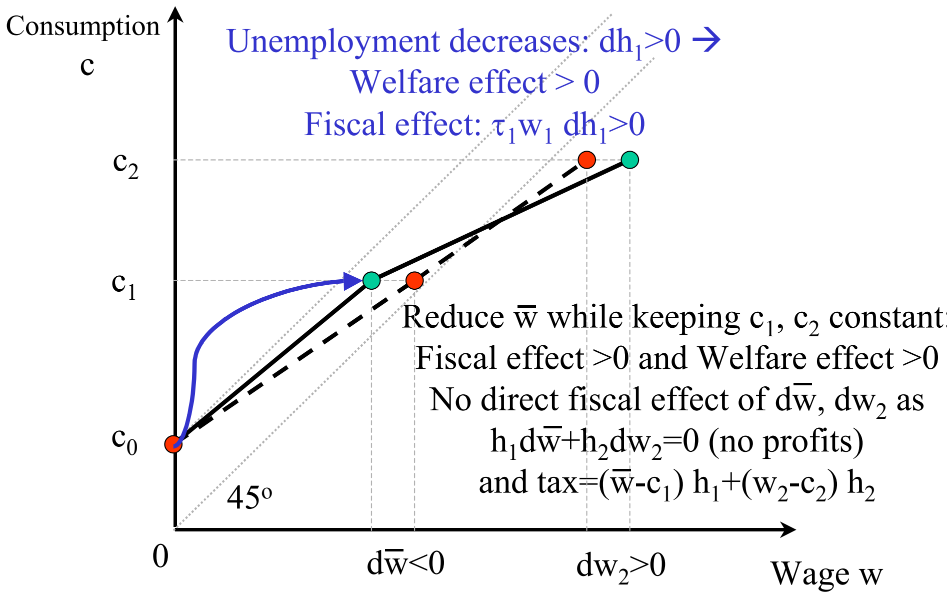

Figure 4. Pareto Improving Policy when $\tau_1>0$ and the minimum wage binds

The Figure starts from a situation with a positive tax rate on low skilled work ($\tau_1>0$) along with a binding minimum wage creating involuntary unemployment. From that situation, consider lowering the minimum wage while keeping c_0, c_1, and c_2 constant. This reform reduces involuntary unemployment, hence increasing welfare of the newly employed and increasing tax revenue as the newly employed pay higher taxes. Therefore, this reform is a Pareto improvement.

Table 1: Optimal Minimum Wage with No Taxes or Fixed Taxes

	σ=0.5 e_1=0.25	σ=0.5 e_1=0.5	σ=0.5 e_1=1	σ=0.25 e_1=0.5	σ=0.5 e_1=0.5	σ=1 e_1=0.5
	(1)	(2)	(3)	(4)	(5)	(6)
A. Optimum Minimum Wage with no taxes and transfers						
A1. Case γ=1						
Minimum Wage / Market Wage	1.12	1.21	1.34	1.44	1.21	1.07
Unemployment Rate	7.6%	16.8%	39.8%	24.8%	16.8%	9.2%
A2. Case γ=3						
Minimum Wage / Market Wage	1.03	1.08	1.17	1.18	1.08	1.03
Unemployment Rate	2.3%	6.9%	20.1%	10.9%	6.9%	3.8%
A3. Case γ=0.5						
Minimum Wage / Market Wage	1.11	1.19	1.29	1.41	1.19	1.06
Unemployment Rate	6.9%	15.0%	34.9%	23.1%	15.0%	8.1%
B. Optimum Minimum Wage with exogenous taxes (uniform tax rate τ=0.35)						
B1. Case γ=1						
Minimum Wage / Market Wage	1.00	1.01	1.04	1.13	1.01	1.00
Unemployment Rate	0.0%	0.5%	5.5%	8.0%	0.5%	0.0%

Notes: The table reports the minimum wage (relative to market wage rate w^*_1) and the induced unemployment rate among the low skilled as a function of the elasticity of substitution σ between low and high skilled labor in production, the elasticity of labor supply of low skilled workers e_1 (the high skilled labor supply elasticity e_2=0.25 in all cases), and the risk aversion γ of the social welfare function.
The production function is CES with elasticity of substitution γ, calibrated so that market equilibrium with no minimum wage is $(w^*_1,w^*_2)=(1,3)$. The supply functions are calibrated so that $(h^*_0,h^*_1,h^*_2)=(.2,.2,.6)$
The social welfare function is such that $G(u)=(u+0.5)^{1-\gamma}/(1-\gamma)$.

Table 2: Optimal Minimum Wage with Optimal Taxes

	σ=0.5	σ=0.5	σ=0.5	σ=0.25	σ=0.5	σ=1
	e_1=0.25	e_1=0.5	e_1=1	e_1=0.5	e_1=0.5	e_1=0.5
	(1)	(2)	(3)	(4)	(5)	(6)
A. Case γ=1						
A1. Optimal Tax Rates with no Minimum Wage						
Tax rate on low skilled workers τ_1	-9.0%	-20.7%	-20.1%	-25.7%	-20.7%	-16.6%
Tax rate on high skilled workers τ_2	45.4%	46.2%	47.6%	47.0%	46.2%	45.4%
A2. Optimal Tax Rates and optimal Minimum Wage						
Tax rate on low skilled workers τ_1	-13.1%	-45.0%	-98.4%	-56.4%	-45.0%	-35.6%
Tax rate on high skilled workers τ_2	45.2%	44.9%	44.2%	45.5%	44.9%	44.3%
Minimum Wage / Market Wage	1.02	1.11	1.31	1.20	1.11	1.06
Unemployment Rate	1.6%	11.2%	52.1%	13.2%	11.2%	9.3%
B. Case γ=3						
B1. Optimal Tax Rates with no Minimum Wage						
Tax rate on low skilled workers τ_1	28.6%	10.2%	-7.6%	5.3%	10.2%	14.1%
Tax rate on high skilled workers τ_2	64.0%	64.2%	64.7%	64.6%	64.2%	63.8%
B2. Optimal Tax Rates and optimal Minimum Wage						
Tax rate on low skilled workers τ_1	28.6%	10.2%	-15.5%	5.3%	10.2%	14.1%
Tax rate on high skilled workers τ_2	64.0%	64.2%	64.4%	64.6%	64.2%	63.8%
Minimum Wage / Market Wage	1.00	1.00	1.05	1.00	1.00	1.00
Unemployment Rate	0.0%	0.0%	6.8%	0.0%	0.0%	0.0%
C. Case γ=0.5						
C1. Optimal Tax Rates with no Minimum Wage						
Tax rate on low skilled workers τ_1	-21.9%	-23.3%	-16.6%	-25.9%	-23.3%	-17.8%
Tax rate on high skilled workers τ_2	32.4%	33.6%	35.2%	34.4%	33.6%	32.0%
C2. Optimal Tax Rates and optimal Minimum Wage						
Tax rate on low skilled workers τ_1	-41.0%	-81.3%	-153.3%	-93.2%	-81.3%	-34.1%
Tax rate on high skilled workers τ_2	31.3%	30.7%	29.1%	31.5%	30.7%	31.1%
Minimum Wage / Market Wage	1.08	1.21	1.49	1.35	1.21	1.04
Unemployment Rate	5.5%	21.9%	91.5%	23.8%	21.9%	4.7%

Notes: The table reports optimal tax rates (on low and high skilled) with no minimum wage and the joint optimal tax rates and minimum wage (relative to market wage rate w^*_1) and the induced unemployment rate among the low skilled as a function of the elasticity of substitution σ between low and high skilled labor in production, the elasticity of labor supply of low skilled workers e_1 (the high skilled labor supply elasticity e_2=0.25 in all cases), and the risk aversion γ of the social welfare function.
The production function is CES with elasticity of substitution γ, calibrated so that market equilibrium with no minimum wage is $(w^*_1,w^*_2)=(1,3)$. The supply functions are calibrated so that $(h^*_0,h^*_1,h^*_2)=(.2,.2,.6)$
The social welfare function is such that $G(u)=(u+0.5)^{1-\gamma}/(1-\gamma)$.