RATIONING AND THE SUPPLY OF LABOR:
AN ECONOMETRIC APPROACH*

John C. Ham
University of Toronto

* This paper is part of my Ph.D. thesis in progress in the Department of Economics, Princeton University. I would like to thank my advisors, Orley Ashenfelter and Richard Quandt for excellent guidance. I would also like to thank James Heckman, Peter Ivanick, Randall Olsen, Albert Rees Harvey Rosen, Daniel Saks, and Donald Sant for helpful discussions, and James Brown, Roger Gordon, Jerry Hausman, Stephen Nickell, and David Wise for valuable comments on an earlier version of this paper. Two anonymous referees also made excellent comments on the paper. Robert Avery and Charles Manski pointed out an important error in the earlier version of the paper. I am solely responsible for any remaining errors. Financial support was generously provided by the Canada Council, the Industrial Relations Section, Princeton University, and a grant to the Princeton University Economics Department from the Sloan Foundation.
The estimation of labor supply functions has been one of the most active areas of economic research in recent years.\(^1\) Although empirical studies have differed in approach, virtually all have been based on the crucial assumption that anyone working a positive number of hours is at a point of tangency between an indifference curve and a budget constraint.\(^2\)

There are at least three groups of workers for which this assumption seems potentially restrictive. The first group consists of the unemployed, who by definition must suffer a period without paid work but are unhappy with their situation. The two other potentially important groups consist of those workers who want to work more hours (the underemployed) or fewer hours (the overemployed) on their job than is currently available. In what follows, I will refer to these workers as rationed.

With the increase in measured unemployment in recent years the potential problems for estimating labor supply functions posed by rationing have become more significant.\(^3\) In this paper I develop an estimation scheme for labor supply functions when rationing is present. Because of increased skepticism on the part of some concerning the extent to which unemployment truly represents a constraint on labor supply behavior, I also present some simple tests of whether this rationing is a real constraint on labor market behavior.

\(^1\) See Killingsworth (forthcoming) for an excellent summary of this research.

\(^2\) Exceptions using aggregate data are Ashenfelter (1977) and Rosen and Quandt (1978). Exceptions using micro data are DeVanzo, DeTray and Greenberg (1976), Hurd (1976), Raet (1974), and Wales and Woodland (1976), (1977). Dickinson (1975) has investigated the non-tangency issue from a theoretical point of view.

\(^3\) The possibility of rationing also presents a serious problem in the theory of equitable taxation. If one believes that an individual should be taxed on the basis of the opportunities he faces (such as his wage), it is important that any additional constraints he faces be incorporated into the tax structure. This point has been noted by Gordon (1976).
The outline of the paper is as follows: Section I presents a very brief summary of the theoretical work on rationing and tabulates longitudinal data from the University of Michigan Panel Study of Income Dynamics (PSID) to determine the potential extent of rationing in the labor market. Section II contains a discussion of the approaches to the rationing problem taken by previous researchers, while Section III offers a new approach to the rationing problem. Section IV tentatively implements the new approach. Section V tests whether those who claim to be rationed truly are. The empirical work of Section IV suggests that ignoring the rationing problem significantly affects the parameter estimates. The test results in Section V, based on the parameter estimates of the labor supply function from the approach introduced here, support the hypothesis that the unemployed are truly rationed. On the other hand, the test results based on the least squares estimates suggest that unmeasured taste differences may be quite important in determining the hours of work of those who claim to be rationed.

I. Is Rationing an Important Problem?

Before rationing in the labor market can be considered a potentially serious problem, two questions must be investigated. First, is there any reason or theoretical justification for believing those who claim to be rationed? Second, does rationing seem to affect a significant portion of the labor force or only a few isolated individuals?

The first question must be answered affirmatively, since there are several explanations for the existence of rationing. For example, an individual may choose a job with random hours on the basis of his long run tastes, so that in any one year he may be overemployed or underemployed,
although in the long run his hours worked in this job should equal his desired labor supply. Alternatively, a worker may prefer to have a part-time job, but the wages on these jobs may be so low compared to those paid on full-time jobs that he reaches a higher level of utility by taking a full-time job and being overemployed.

Moreover, in recent years several authors have produced theoretical models consistent with the existence of rationing in the labor market. For example, first Clower (1965) and then Barro and Grossman (1971) elaborate on a Keynesian model in which disequilibrium leads to rationing in the labor market. In these models the rationing is unanticipated. Feldstein (1976) and Baily (1977) present models where unemployment insurance and voluntary agreements between workers and management produce temporary layoffs and underemployment. In the work of Baily (1974), Gordon (1974) and Azariadis (1975) differences in risk aversion between workers and management lead to temporary layoffs. In all these models rationed workers are constrained in the sense that they would like to work more hours at their given wage, albeit they also voluntarily entered the job knowing that they were likely to be constrained in certain periods of the business cycle. In the wage contract models workers are compensated for the potential unemployment or underemployment through a higher wage (and unemployment insurance) and thus have no incentive to search for jobs.

4/ The constant wage models of Baily (1974) and Azariadis are most compatible with the empirical work carried out below. In these models the wage will not vary and thus workers will form their desired labor supply on the basis of this wage. It should be noted that in the variable wage model of Baily (1977) or the Lucas and Rapping (1969) model intertemporal considerations become important, and in estimating a one period model I may be committing a specification error by ignoring wages expected in the future.

5/ For example, Feldstein states "Any particular spell of unemployment may be both involuntary and loudly protested, even though the decision rule that led to the layoff may have been chosen by the employees." (1976, p. 955).

6/ Abowd and Ashenfelter (1979) estimate such compensating differentials for rationed workers.
without constraints. Whether or not workers have anticipated the rationing or have been compensated for it, they are not employed for the number of hours that would be implied by an unconstrained labor supply function.

In order to answer the second question, and investigate how many individuals may be affected by rationing, a sample of 861 males between the ages of 25 and 50 in 1967 was taken from the PSID. Each year the PSID contained retrospective questions on whether there was more work available (in the previous year) on a worker's job (any of his jobs) so that he could have worked more if he had wanted to, and if not, whether he had wanted to work more. I am assuming that an individual who answers no to the first question and yes to the second had desired hours of work greater than the hours he actually worked and thus was classified for the purposes of this study as underemployed. The PSID also asked whether an individual could have worked fewer hours on any of his jobs, and if not, whether he had wanted to work less even if he would have earned less money. I assumed that an individual who answered no to the first question and yes to the second had desired hours less than actual and thus was classified as overemployed. An individual was classified as unemployed if he stated that he lost one or more days of work that year because of unemployment. Table 1 contains a breakdown of the sample by their employment experiences for the eight-year period 1967-1974.

The data contained in Table 1 are quite interesting. The percentage of the sample rationed in any one year ranges from a low of 23% in 1967 to a high of 34% in 1970. Even more striking is the fact that 73% of the sample is rationed at least once over the eight-year period. Thus rationing
### Table 1

Employment Experience of Males

(Aged 25-50 in 1967) For the Years 1967-1974

<table>
<thead>
<tr>
<th>Year</th>
<th>Not Rationed</th>
<th>Underemployed</th>
<th>Unemployed</th>
<th>Overemployed</th>
<th>Underemployed and Unemployed*</th>
<th>Overemployed and Rationed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967</td>
<td>76.66</td>
<td>12.54</td>
<td>11.03</td>
<td>3.48</td>
<td>3.46</td>
<td>0.23</td>
</tr>
<tr>
<td>1968</td>
<td>71.89</td>
<td>15.80</td>
<td>9.76</td>
<td>6.16</td>
<td>3.37</td>
<td>0.23</td>
</tr>
<tr>
<td>1969</td>
<td>70.96</td>
<td>18.00</td>
<td>9.87</td>
<td>5.11</td>
<td>3.60</td>
<td>0.35</td>
</tr>
<tr>
<td>1970</td>
<td>65.97</td>
<td>19.74</td>
<td>13.82</td>
<td>6.62</td>
<td>5.57</td>
<td>0.58</td>
</tr>
<tr>
<td>1971</td>
<td>68.64</td>
<td>17.77</td>
<td>11.96</td>
<td>6.04</td>
<td>3.95</td>
<td>0.46</td>
</tr>
<tr>
<td>1972</td>
<td>72.82</td>
<td>14.17</td>
<td>8.48</td>
<td>7.20</td>
<td>2.21</td>
<td>0.46</td>
</tr>
<tr>
<td>1973</td>
<td>75.96</td>
<td>13.12</td>
<td>8.83</td>
<td>5.11</td>
<td>2.67</td>
<td>0.35</td>
</tr>
<tr>
<td>1974</td>
<td>67.25</td>
<td>17.65</td>
<td>14.29</td>
<td>5.81</td>
<td>4.76</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Percent of Sample Never Rationed: 27.87
Percent of Sample Never Underemployed or Overemployed: 35.19
Percent of Sample Never Unemployed: 63.18
Percent of Sample Always Rationed: 1.27
Total Sample: 861

*Any individual in this column has also been included in the underemployed column and unemployed column—thus total sample = number not rationed + number underemployed + number unemployed + number overemployed = (number underemployed and unemployed) - (number overemployed and unemployed).
may affect a large portion of the labor force and is potentially too serious a problem to simply assume away.

II. Past Approaches to Rationing

In the past there have been three approaches to rationing: to ignore it; to truncate the sample on the basis of it; or to adjust hours of work for it. The first approach is by far the most common and has been used in almost every labor supply study. To see the kind of econometric problems that arise in this situation consider the case of an individual who is truly unemployed or underemployed. Letting \( H_i^d, w_i, y_i \) and \( R_i \) be the desired hours of work, wage rate, unearned income, and a vector of taste variables for individual \( i \), write his labor supply function as

\[
H_i^d = h(w_i, y_i, R_i) + \varepsilon_i, \tag{1}
\]

where \( \varepsilon_i \) is an error term representing omitted variables and an approximation error resulting from assuming that all individuals have the same supply function. In what follows it will be assumed that \( \varepsilon_i \) is homoskedastic and independent of \( w_i, y_i \) and \( R_i \). Next, define the vector \( X_i' = (1, w_i, y_i, R_i') \) and assume for the sake of simplicity that \( h(\cdot) \) is linear in its arguments. Then rewrite (1) as

\[
H_i^d = X_i' \beta + \varepsilon_i. \tag{2}
\]

Equation (2) can of course be estimated by standard regression techniques given the necessary data. However, for the unemployed and underemployed only actual hours \( H_i \) are available. Now by definition \( H_i < H_i^d \), which
implies that $H_1 - H_1^d < 0$. Equation (2) may then be rewritten as

$$H_1 = X_1' \beta + \epsilon_1 + (H_1 - H_1^d) = X_1' \beta + u_1.$$  \hfill (3)

If the rationing is random, then using $H_1$ instead of $H_1^d$ as a dependent variable will simply lower the constant term. On the other hand, if the rationed are more likely to have low wages, low unearned income, or little education as is usually thought to be the case, then the independent variables will be correlated with the new error term $u_1$, and the resulting parameter estimates will be inconsistent.

An alternate approach to the rationing problem is to remove the rationed from the sample. Wales and Woodland (1976), (1977) removed the overemployed and underemployed from the sample, while DaVanzo, De Tray and Greenberg (1976) removed the unemployed from the sample.

This approach to rationing has at least two serious drawbacks. First, it wastes information. For almost one-third of the PSID sample in any year it would be necessary to discard the inequality information available on $H_1^d$. The method proposed in Section III exploits this information. Second, and perhaps of more importance, is the fact that if the probability of being rationed is correlated with $\epsilon_1$ and the independent variables, truncating the rationed from the sample will lead to inconsistent parameter estimates.  

Finally, consider the procedure of adjusting the dependent variable to account for the effects of rationing. For example, Hurd (1976) defined labor supply as hours worked plus unemployment. If, however, some of the

\[\text{See Hausman and Wise (1977) and Heckman (1979) for discussions of the truncation and censoring issues.}\]
unemployment is leisure, or even if the unemployment is involuntary but
partially compensated for by working harder during the rest of the year,
then Hurd's treatment will add a positive error term to the right hand side
of the labor supply equation. Since this new error term may be correlated
with independent variables such as the wage and unearned income, there is
again the danger of inconsistent parameter estimates.

Clearly all of the above approaches to rationing possess serious
drawbacks. The maximum likelihood approach to rationing introduced in the
next section takes the rationing into account, avoids the censoring problem,
and allows for some of the unemployment to be voluntary.

III. A New Approach to the Rationing Problem

In this section a new approach to the rationing problem is introduced.
This approach is based on a conceptually simple idea: the only information
available concerning the desired hours of work for the rationed is
inequality information. Those who are unemployed or underemployed desire
more hours of work while those who are overemployed desire to work fewer hours.
Through the use of maximum likelihood techniques it is possible to use this
inequality information to estimate the parameters of the labor supply function.¹²/

Consider the following statistical model. Assume each individual \( i \)
has a lower and upper constraint, \( H_{Li} \) and \( H_{ui} \), on the hours he can work.
If his desired hours lie between the two constraints, then we observe his

¹²/ An anonymous referee suggested an alternative to my approach which provides
an interesting avenue for future research. Rather than asking (as I do) "how
many hours does the individual want work given his wage?" one could ask "at
what wage would his desired hours equal his current hours of work?"
desired hours, otherwise we observe one of the constraints. This may be put more formally in a two-limit Tobit model, where \( H_i \) is actual hours:

\[
\begin{align*}
\text{not rationed} & \quad H_i = X_i'\beta + \epsilon_i \quad \text{RHS} \leq \bar{H}_i \leq \underline{H}_i \\
\text{overemployed} & \quad H_i = \bar{H}_i \\
\text{underemployed or unemployed} & \quad H_i = \underline{H}_i \\
\end{align*}
\]

Now assuming \( \epsilon_i \sim \text{iid} \ N(0, \sigma^2) \), it is a straightforward matter to form the likelihood function. The probability of an individual being unemployed or underemployed may be written

\[
P(X_i'\beta + \epsilon_i > \bar{H}_i) = 1 - F\left(\frac{\bar{H}_i - X_i\beta}{\sigma}\right) \tag{5a}
\]

where \( F(\cdot) \) is the cumulative distribution function of the standard normal random variable. The probability of an individual being overemployed may be written

\[
P(X_i'\beta + \epsilon_i < \underline{H}_i) = F\left(\frac{\underline{H}_i - X_i\beta}{\sigma}\right) \tag{5b}
\]

Of course, one cannot observe \( \bar{H}_i \) or \( \underline{H}_i \) directly, but for an individual who is unemployed or underemployed \( \bar{H}_i \) equals actual hours \( H_i \), and for an overemployed individual \( \underline{H}_i \) again equals actual hours \( H_i \). Thus, in this framework it is only necessary to assume that some and not all of the unemployment is involuntary, since all that is assumed is that desired hours are greater than actual hours for the unemployed.

For an unconstrained individual the density function is simply

\[
\frac{1}{\sigma} \cdot f\left(\frac{H_i - X_i\beta}{\sigma}\right), \quad \text{where} \quad f(\cdot) \quad \text{is the density function of the standard normal.}
\]

\[\text{2}/\text{The reader should note that} \quad \bar{H}_i \quad \text{and} \quad \underline{H}_i \quad \text{have been implicitly assumed exogenous for the rationed or in other words, the firm sets the hours of work for these individuals. The author is currently working on an estimation procedure which relaxes this assumption.}\]
Letting $U$ denote the set of all unemployed and underemployed individuals, $O$ the set of all overemployed individuals, and $N$ the set of all non-rationed individuals, the complete likelihood function may then be written as

$$L = \prod_{i \in U} (1-F(Z_i)) \prod_{i \in O} F(Z_i) \prod_{i \in N} \frac{f(Z_i)}{\sigma}$$

(6)

and its logarithm as

$$L^* = \sum_{i \in U} \ln(1-F(Z_i)) + \sum_{i \in O} \ln F(Z_i)$$

$$+ \sum_{i \in N} (-\ln \sigma - 1/2 Z_i^2 + k)$$

(7)

where $Z_i = (H_i - X_i' \beta)/\sigma$ and $k = 1/2 \ln(2\pi)$. Since equation (7) is highly nonlinear, it is necessary to use numerical methods in obtaining maximum likelihood estimates of $\beta$ and $\sigma^2$.

IV. Estimating Labor Supply Functions Under Conditions of Rationing

In order to test the computational feasibility of an estimator based on maximizing (7) and to see whether these estimates would differ appreciably from those based on more conventional approaches a cross-section of 857 prime-aged males was chosen from the Income Dynamics Survey (TUID) for 1971. An individual was included in the sample if:

1. He was either black or white and between the ages of 25 and 50 in 1967.

2. He was not a member of the nonrandom poverty sub-sample in 1967.

3. He stayed in the sample for all of the eight years 1967-1974.

4. He had not retired from the labor force.

5. He was not overemployed and unemployed in the same year.
Criterion 4 was adopted out of expediency and eliminated 34 individuals from the sample. It is clearly an example of truncating on the basis of a dependent variable, although it is a truncation of less than four per cent of the sample.\(^\text{10}\)

Criterion 5 was introduced because desired hours may be greater or less than actual hours for someone who was both unemployed and overemployed in the same year. If the rationing is not random, this could lead to a nonrandom missing data problem. Fortunately, only four individuals were eliminated from the sample on the basis of criterion 5.

Once the sample had been chosen, it was necessary to decide which variables should be included in the labor supply equation. Based on previous labor supply studies such as those contained in Cain and Watts (1973) the following variables were chosen:\(^\text{11}\)

1. after tax net wage
2. net unearned income
3. dummy variable coded 1 if the individual suffered from a health limitation
4. dummy variable coded 1 if the individual was married

\(^\text{10}\) The proper procedure in this case is to write the likelihood (6) as being conditional on \(H_1^d \geq 0\). In some earlier work using Rosen's (1976) tax procedure, I conditioned both (6) and the least-squares likelihood function in this way and the parameter estimates differed little from the unconditional ones. Hurd (1976) also found this result in his work with males.

\(^\text{11}\) The reader should note that these variables were chosen only as factors influencing a worker's desired labor supply. No attempt is being made to model the determination of hours worked under involuntary unemployment or wage contract theory. Thus, the independent variables chosen are independent of the underlying cause of the rationing.
5. age
6. education (in years)
7. dummy variable coded 1 if the individual was black
8. number of children

The use of the after-tax wage and the after-tax unearned income raises the possibility of simultaneous equation bias since both depend on hours of work and are thus endogenous. Moreover, the gross wage was formed by dividing earnings by hours worked and consequently the gross wage may be correlated with the error term in the labor supply equation. As a partial solution to this problem I replaced the after-tax wage and unearned income with predicted values from auxiliary regressions.\textsuperscript{12}/ I used the age-squared, education-squared, age times education, the after-tax wage and unearned income from the previous year, as well as the other explanatory variables in the labor supply equation as independent variables in these regressions.\textsuperscript{13}/ (Seperate wage and unearned income equations were estimated for each sample used in the labor supply equations in Table 2.) The use of the lagged dependent variables as instruments clearly creates some problems; however this approach appeared better than using the actual after-tax variables in the labor supply estimation or using only the age-education terms to identify the equation. The latter procedure was not explored, since such equations were considered likely to have very poor explanatory power.

\textsuperscript{12}/ This approach is also appropriate if the worker changes jobs and faces two different wage rates during the year. Use of the mean wage would not create too many problems (because of the linearity of the labor supply function) but to the extent that measurement error is compounded in this situation, using the predicted values will alleviate the problem.

\textsuperscript{13}/ The lagged dependent variables dominated their respective equations. The values of R\textsuperscript{2} were .422 in the wage equation and .586 in the unearned income equation when the full sample was used for estimation. The parameter estimates for these equations are available from the author.
There remains one problem for correction in future work. By not including the wife's wage in the supply equation, I am ignoring any family labor supply considerations. Handling this problem correctly is quite difficult, as the labor supply function would differ between husbands with working and nonworking wives and whether or not the wife works will be correlated with the error term in the husband's supply function. Olsen (1977) contains a complete discussion of this problem and a procedure for handling it.

Turn first to the basic results of estimating the labor supply function by least squares for the full, non-rationed and rationed samples that have been placed in the first three columns of Table 2. The dependent variable for all equations (including the maximum likelihood one below) is annual hours worked—it is not hours in the labor force. From these results it is clear that regressions on the rationed and non-rationed samples produce somewhat different coefficients. A Chow test performed on the significance of this difference produced a calculated F-ratio of 4.413. This ratio is considerably larger than the critical value of F (9.837) = 2.46 at the .01 level, and leads to the conclusion that these sets of coefficients are significantly different.

The maximum likelihood estimates based on (7) are contained in the fourth column of Table 2.\(^ {1k} \) The maximum likelihood coefficients (MLC), the full sample regression coefficients (FRC) and the non-rationed regression coefficients (NRC) are all quite close for the wage, health limitation dummy

\(^ {1k} \) The DFP algorithm described in Goldfeld and Quandt (1972, chapter 1) was used to maximize the likelihood function.
<table>
<thead>
<tr>
<th>Coefficient of</th>
<th>Least Squares (Full Sample)</th>
<th>Least Squares (Non-Rationed)</th>
<th>Least Squares (Rationed)</th>
<th>Maximum Likelihood (Full Sample)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>2.413</td>
<td>2.485</td>
<td>2.572</td>
<td>3.002</td>
</tr>
<tr>
<td></td>
<td>(.1821)</td>
<td>(.1696)</td>
<td>(.3954)</td>
<td>(.2052)</td>
</tr>
<tr>
<td>Wage ($)</td>
<td>-.1801</td>
<td>-.1791</td>
<td>-.1575</td>
<td>-.1616</td>
</tr>
<tr>
<td></td>
<td>(.0181)</td>
<td>(.0197)</td>
<td>(.0373)</td>
<td>(.0274)</td>
</tr>
<tr>
<td>Unearned income (0000's of $)</td>
<td>.7959</td>
<td>.7136</td>
<td>.7682</td>
<td>.5968</td>
</tr>
<tr>
<td></td>
<td>(.1140)</td>
<td>(.1121)</td>
<td>(.3145)</td>
<td>(.1199)</td>
</tr>
<tr>
<td>Race dummy</td>
<td>-.1489</td>
<td>-.1430</td>
<td>-.1281</td>
<td>-.0896</td>
</tr>
<tr>
<td></td>
<td>(.0838)</td>
<td>(.0954)</td>
<td>(.1641)</td>
<td>(.0963)</td>
</tr>
<tr>
<td>Number of children</td>
<td>-.0013</td>
<td>.0060</td>
<td>-.0539</td>
<td>.0197</td>
</tr>
<tr>
<td></td>
<td>(.0132)</td>
<td>(.0139)</td>
<td>(.0296)</td>
<td>(.0147)</td>
</tr>
<tr>
<td>Health limitation dummy</td>
<td>-.3311</td>
<td>-.2869</td>
<td>-.4171</td>
<td>-.3032</td>
</tr>
<tr>
<td></td>
<td>(.0859)</td>
<td>(.0906)</td>
<td>(.1866)</td>
<td>(.0953)</td>
</tr>
<tr>
<td>Marriage dummy</td>
<td>.1964</td>
<td>.1844</td>
<td>.2231</td>
<td>1.663</td>
</tr>
<tr>
<td></td>
<td>(.0970)</td>
<td>(.1018)</td>
<td>(.2171)</td>
<td>(.1062)</td>
</tr>
<tr>
<td>Age</td>
<td>-.0462</td>
<td>-.0555</td>
<td>-.0705</td>
<td>-.0968</td>
</tr>
<tr>
<td></td>
<td>(.0305)</td>
<td>(.0319)</td>
<td>(.0677)</td>
<td>(.0383)</td>
</tr>
<tr>
<td>Education</td>
<td>.0396</td>
<td>.0850</td>
<td>.0048</td>
<td>.0144</td>
</tr>
<tr>
<td></td>
<td>(.0075)</td>
<td>(.0090)</td>
<td>(.0065)</td>
<td>(.0083)</td>
</tr>
<tr>
<td>R²</td>
<td>.175</td>
<td>.318</td>
<td>-.132</td>
<td>--</td>
</tr>
<tr>
<td>S.E.E.</td>
<td>.589</td>
<td>.5122</td>
<td>.6919</td>
<td>.5924</td>
</tr>
<tr>
<td>Sample size</td>
<td>855 b/</td>
<td>589</td>
<td>266</td>
<td>855</td>
</tr>
</tbody>
</table>

Dependent Variable: Annual Hours Worked (in 000's).

a/ Standard errors in parenthesis - asymptotic standard errors for maximum likelihood estimates.

b/ I excluded 4 individuals both overemployed and unemployed in 1971 and 2 individuals with earnings less than $1 in 1970 or 1971.

c/ In the regressions the R² and S.E.E. were calculated using the actual values of the wage and unearned income.
and the marital dummy. The wage coefficients imply an elasticity (evaluated at the sample means) of -.28, reasonably close to Lewis' (1974) estimate of -.15 for the long-run wage elasticity. The marital status coefficients are all positive and almost significant, while the health limitation coefficients are all large, negative, and significantly different from zero.

The situation with respect to the remaining six coefficients is quite different. The MLC of the constant is substantially larger than the FRC, indicating that ignoring the rationing problem does indeed lower the constant. The MLC for unearned income is almost two standard errors smaller than the FRC. All of the unearned income coefficients are positive, with an estimated income elasticity of .120 from the FRC and .090 from the MLC. Since these estimates imply that leisure is an inferior good, these coefficients must be considered unsatisfactory and an issue for future research.15/

The FRC for race is almost significant while the MLC for this variable is clearly not. The MLC for education is not quite significant and is approximately three standard errors lower than the FRC. Thus race and education become less important in labor supply analysis once rationing is accounted for.

The opposite situation occurs with respect to age and number of children. The MLC for age is twice as large as the FRC and is also much more significant. The MLC for children is over ten times as large as the

12/ This problem has plagued many other labor supply studies, for example, see the results of Rosen and Quandt (1978) and their discussion of previous work on page 376. Greenberg and Koster (1973) provide a possible explanation for this phenomenon. They argue that unearned income may be positively correlated with the labor supply error term since those who work long hours are likely to have a high taste for assets and hence a high level of unearned income. Since the predicted value for unearned income used here depends on past unearned income which depends on past labor supply, it will not correct this problem.
FRC, and the MLC is almost significant while the FRC is very insignificant.

Thus it makes an important difference to labor supply analysis which estimation method is used. Of course, it would be very useful to know if the rationed model is the correct one and if those who claim to be rationed truly are. The paper now turns to this issue.

V. Testing for Rationing

In this section two simple tests for rationing are proposed.\(^{16}\)

One test calculates the proportion of those claiming to be rationed who have predicted hours greater than actual. In the absence of rationing this proportion should not differ significantly from .5, but if the underemployed and unemployed (overemployed) truly are constrained the proportion should

\(^{16}\) In an earlier version of this paper I had attempted to test for rationing by assuming that \(\lambda\) of those who claimed to be unemployed or underemployed were truly rationed, and thus the likelihood function for an unemployed individual becomes \(\lambda(1 - F(Z_1)) + (1 - \lambda) f(Z_1)/\sigma\). I then argued that \(\lambda\) could be estimated along with the other parameters by maximum likelihood. Unfortunately, while \(Z_1\) is invariant to the units chosen, \(\sigma\) is not and thus the estimate of \(\lambda\) will depend on the units. This problem disappears if one notes that the likelihood function for this individual should be written as \(\lambda(1 - F(Z_1)) + (1 - \lambda) f(Z_1)/\sigma dH_1\), where \(Z_1 = (H_1 - X_1\beta)/\sigma\). Unfortunately the second term is now almost zero and the maximum likelihood estimate of \(\lambda\) is one.
be significantly greater (less) than .5. This second test measures the statistical significance of the mean difference between predicted and actual hours for those claiming to be rationed. If the underemployed and unemployed (overemployed) are rationed, this difference should be significantly greater (less) than 0. The second test also allows one to obtain a confidence interval on the number of hours these individuals are rationed.

Before performing these tests one must choose a set of estimates of the labor supply function to predict desired hours. I used both the maximum likelihood estimates and the least squares estimates to predict desired hours. One must also choose a sample for prediction, and using the same sample for prediction and estimation creates a problem. Because the likelihood function is maximized by estimating predicted hours to be as large (small) as possible.

This result is an asymptotic one. Consider the case where there is no rationing and actual hours \( H = X\hat{\beta} + \epsilon \). Then \( P(X\hat{\hat{\beta}} > H) = P(X\hat{\hat{\beta}} > X\hat{\beta} + \epsilon) = P(X(\hat{\hat{\beta}} - \hat{\beta}) > \epsilon) \approx P(0 > \epsilon) = .5 \), since \( \epsilon \) is distributed \( N(0, \sigma^2) \), and as the sample size approaches infinity, \( (\hat{\beta} - \beta) \) approaches zero. For the case where the individual is truly rationed, and \( H < X\hat{\beta} + \epsilon \), we have \( P(X\hat{\hat{\beta}} > H) = P(X\hat{\hat{\hat{\beta}}} > X\hat{\beta} + \epsilon) = .5 \). A similar result follows for the overemployed.

In interpreting these tests two points should be noted. First, a very simple labor supply function was estimated; in particular intertemporal considerations were ignored. Second, the existence of rationing is consistent with both Keynesian theory and contract theory, and this must be kept in mind when drawing normative implications from the analysis.
for the underemployed and unemployed (overemployed) individuals, using
the same data for estimation and prediction would implicitly bias the
test in favor of accepting the rationing hypothesis. As a partial
solution to this problem I have carried out the prediction analysis for
the year preceding (1970) and following (1972) the period used for
estimation.\textsuperscript{19} While this procedure does have the advantage of using
different observations for estimation and prediction, it should only
be considered a partial solution since the new data consists of more
observations on the same individuals that were used for estimation.

Table 3 presents data on the prediction performance of the maximum
likelihood estimates of the labor supply equation for the different
labor force groups in 1970 and 1972.\textsuperscript{19} Consider first sections 1 and 2
of the table. I have assumed that each comparison of predicted and actual
hours represents an independent binomial trial. This assumption allows
me to form a standard error for the proportion $p_i$, in group $i$, with pre-
dicted hours greater than actual. However, this assumption is clearly
unrealistic, since the same estimated $\hat{\beta}$ is used for each comparison. The
use of the independence assumption simply reflects the lack of a tractable
alternative for calculating the standard errors.

For the overemployed group, the proportion $p_i$ is quite close to,
and not significantly different from, .5 in both 1970 and 1972.\textsuperscript{20} In each
of these years the mean difference between predicted and actual hours (hereafter

\textsuperscript{19} The actual after tax wage and unearned income for these years were used in
the prediction.

\textsuperscript{20} Conditional on the assumption of independent binomial trials, the variance
of $p_i$ is $p_i(1-p_i)/n_i$, where $n_i$ is the number of individuals in group $i$. 
<table>
<thead>
<tr>
<th>Group</th>
<th>Number</th>
<th>Proportion with ( \hat{n} &gt; \bar{n} )</th>
<th>Mean (( \hat{n} - \bar{n} ))</th>
<th>Difference Between Rationed and Non-Rationed Workers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( n_1 )</td>
<td>( \hat{p}_1 )</td>
<td>Mean (( \hat{n} - \bar{n} ))</td>
<td>( \hat{p}_1 )</td>
</tr>
<tr>
<td>1. 1970 - Entire Sample</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overemployed</td>
<td>52</td>
<td>.558</td>
<td>-30.7</td>
<td>.005</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( .069 )</td>
<td>( 86.6 )</td>
<td>( .072 )</td>
</tr>
<tr>
<td>Under. &amp; not Unem.</td>
<td>122</td>
<td>.977</td>
<td>311.8</td>
<td>.314</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( .030 )</td>
<td>( 62.4 )</td>
<td>( .036 )</td>
</tr>
<tr>
<td>Unem. &amp; not Under.</td>
<td>66</td>
<td>.848</td>
<td>366.7</td>
<td>.286</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( .044 )</td>
<td>( 78.7 )</td>
<td>( .049 )</td>
</tr>
<tr>
<td>Unem. &amp; Under.</td>
<td>48</td>
<td>.958</td>
<td>632.2</td>
<td>.396</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( .029 )</td>
<td>( 91.8 )</td>
<td>( .036 )</td>
</tr>
<tr>
<td>Not Rationed</td>
<td>567</td>
<td>.553</td>
<td>1.4</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( .021 )</td>
<td>( 34.5 )</td>
<td>---</td>
</tr>
<tr>
<td>2. 1972 - Entire Sample</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overemployed</td>
<td>58</td>
<td>.379</td>
<td>-110.3</td>
<td>-.107</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( .064 )</td>
<td>( 82.5 )</td>
<td>( .067 )</td>
</tr>
<tr>
<td>Under. &amp; not Unem.</td>
<td>103</td>
<td>.767</td>
<td>180.4</td>
<td>.281</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( .042 )</td>
<td>( 65.5 )</td>
<td>( .046 )</td>
</tr>
<tr>
<td>Unem. &amp; not Under.</td>
<td>50</td>
<td>.700</td>
<td>331.3</td>
<td>.214</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( .066 )</td>
<td>( 90.0 )</td>
<td>( .068 )</td>
</tr>
<tr>
<td>Unem. &amp; Under.</td>
<td>18</td>
<td>.889</td>
<td>548.2</td>
<td>.402</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( .074 )</td>
<td>( 143.5 )</td>
<td>( .077 )</td>
</tr>
<tr>
<td>Not Rationed</td>
<td>627</td>
<td>.686</td>
<td>-60.3</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( .020 )</td>
<td>( 34.5 )</td>
<td>---</td>
</tr>
<tr>
<td>3. 1970 - Unemployed or Underemployed in 1971 (K71)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Under. &amp; not Unem.</td>
<td>83</td>
<td>.867</td>
<td>329.5</td>
<td>.196</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( .037 )</td>
<td>( 72.8 )</td>
<td>( .069 )</td>
</tr>
<tr>
<td>Unem. &amp; not Under.</td>
<td>28</td>
<td>.857</td>
<td>333.8</td>
<td>.185</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( .066 )</td>
<td>( 115.8 )</td>
<td>( .068 )</td>
</tr>
<tr>
<td>Unem. &amp; Under.</td>
<td>36</td>
<td>.972</td>
<td>681.5</td>
<td>.300</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( .027 )</td>
<td>( 106.1 )</td>
<td>( .065 )</td>
</tr>
<tr>
<td>Not Rationed</td>
<td>64</td>
<td>.672</td>
<td>109.2</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( .059 )</td>
<td>( 79.0 )</td>
<td>---</td>
</tr>
<tr>
<td>4. 1972 - Unemployed or Underemployed in 1971 (K71)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Under. &amp; not Unem.</td>
<td>60</td>
<td>.700</td>
<td>167.2</td>
<td>.042</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( .059 )</td>
<td>( 82.7 )</td>
<td>( .074 )</td>
</tr>
<tr>
<td>Unem. &amp; not Under.</td>
<td>29</td>
<td>.690</td>
<td>346.5</td>
<td>.032</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( .086 )</td>
<td>( 115.8 )</td>
<td>( .097 )</td>
</tr>
<tr>
<td>Unem. &amp; Under.</td>
<td>13</td>
<td>.846</td>
<td>509.5</td>
<td>.186</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( .100 )</td>
<td>( 165.0 )</td>
<td>( .110 )</td>
</tr>
<tr>
<td>Not Rationed</td>
<td>111</td>
<td>.658</td>
<td>99.1</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( .045 )</td>
<td>( 63.1 )</td>
<td>---</td>
</tr>
</tbody>
</table>

\( a/ \) \( \hat{n} \) refers to predicted hours while \( \bar{n} \) refers to actual hours.

\( b/ \) Excludes workers unemployed and overemployed in same year.

\( c/ \) Under. refers to underemployed while Unem. refers to unemployed.

\( d/ \) Standard errors in parenthesis.
mean difference) has the expected sign but is not significantly different from zero at conventional test levels.\textsuperscript{21} Therefore it appears that the overemployed were not truly rationed on these years.

In 1970 the proportion $P_{i}$ with predicted hours greater than actual is quite large and very significantly different from .5 for the three unemployed/underemployed groups. The mean differences for these individuals are large, and the hypotheses that these differences equal 0 are clearly rejected. In 1972 the proportions $P_{i}$ for these workers are lower than in 1970, but are still significantly different from .5. The mean difference for the underemployed in 1972 falls to almost one-half of the mean difference for the unemployed, but all three mean differences are significantly different from zero in this year.

\textsuperscript{21} Let $\bar{X}_{i}$ be the mean of the $X$ variables for individuals in group $i$. Then the estimated variance of the difference between mean predicted and actual hours is $\bar{X}_{i} \cdot V \bar{X}_{i} + \hat{\sigma}^2/n_{i}$, where $V$ is the estimated variance covariance matrix of $\hat{\theta}$, $\hat{\sigma}^2$ is the estimated variance of the error term in the desired hours equation, and $n_{i}$ is the number of individuals in group $i$. 
As a consistency check, I have also carried out the tests for the non-rationed. In 1970 the proportion \( P_i \) is quite close to .5, but because its standard error is so small, is also significantly greater than .5. In 1972 the proportion \( P_i \) is estimated at less than .5 for these workers. The mean difference for the non-rationed is only 1 hour in 1970 and is negative in 1972. Thus the test results for the unemployed/underemployed individuals were not due to the estimated equation over-predicting hours.

Additional evidence is supplied by testing the significance of the difference in the proportions \( P_i \) and the mean differences (between actual and predicted hours) for the constrained groups and the non-constrained group. The estimates of the differences between the constrained and unconstrained workers have been placed in columns 3 and 4 of the Table. As can be seen from the Table, in both 1970 and 1972 proportions \( P_i \) and the mean differences for the unemployed/underemployed groups are significantly greater than those for the non-rationed.

The results from sections 1 and 2 of the table would appear to provide support for the hypothesis that the unemployed and underemployed were rationed in 1970 and 1972. There is, however, another explanation of these results. Those who claim to be rationed may

---

\[ 22^\text{a} \] I have calculated the variance of \( P_i - P_j \) as the sum of the variance of \( P_i \) and the variance of \( P_j \). I used the expression \( (\bar{x}_i - \bar{x}_j)\text{'s}^2 + \sigma^2(1/n_i + 1/n_j) \) to calculate the variance of the difference in the mean difference (between predicted and actual hours) between group \( i \) and \( j \).

(See footnotes 20 and 21 for an explanation of the notation used.)
posses a higher taste for leisure which could account for their low hours of work in column 6 of the table. To explore this issue, I have repeated the prediction analysis for those unemployed or underemployed in 1971 (hereafter the R71 group).

Consider the workers claiming to be constrained in 1971 but not constrained in 1970. If these workers do have a higher taste for leisure, then the proportion $P_1$ of them with predicted hours greater than actual should be greater than .5 in 1970, even though they said they were not constrained in this year. Further, if they really do have a higher taste for leisure, the mean difference between predicted and actual hours should be significantly greater than 0 in 1970. From section 3 of the Table it is clear that the group constrained in 1971 but not constrained in 1970 does indeed have a proportion $P_1$ significantly greater than .5 in 1970, although their mean difference in this year is not significantly different from 0. In 1972 the results for those constrained in 1971 but not constrained in 1972 follow the same pattern — the proportion $P_1$ is significantly different from .5 but the mean difference is not significantly different from 0.

Since these test results indicate that within the R71 group the non-rationed workers and rationed workers are acting in a similar manner, it is important to test whether there is any difference between the two groups. The difference in the respective proportions $P_1$ and the mean difference between predicted and actual hours for those within the R71 category who do not claim to be rationed have been placed in columns 4 and 5 of the table.
In 1970 all of the proportions $p_i$ for the R71 unemployed-underemployed individuals are significantly higher than those for the R71 non-rationed, while in 1972 they are not. In 1970 the mean difference between predicted and actual hours for the underemployed is significantly greater than that for the non-rationed. The mean difference for the underemployed is also significantly higher than that for the non-rationed (assuming a one-tail test at the .05 level). In 1972 the mean difference for the underemployed is not significantly higher than the mean difference for the non-rationed, while the opposite is true for the unemployed.

The results of sections 3 and 4 of the Table indicate that unmeasured tastes may play an important role in explaining the hours of work of those who claim to be rationed, but the data indicate that it is unlikely that tastes explain all of the difference in hours worked between the unemployed and non-constrained workers. The only comparison in all of Table 3 in which the unemployed do not differ substantially from the non-rationed group is in the R71 1972 comparison of the proportion $p_i$. In my view, this is evidence in favor of the hypothesis that the unemployed were truly rationed in 1970 and 1972. The evidence supporting the hypothesis that the underemployed were truly rationed is weaker, and the data from section 4 of Table 3 indicate that unmeasured taste differences may explain much of their relatively low hours of work.
To investigate the robustness of the above results, I have repeated
the analysis using the least squares estimates to predict desired hours.
The new test results are in Table 4. The data contained in section 1 of
Table 4 support the rationing hypothesis, as do those results contained
in section 2 that refer to the difference between the rationed and non-
rationed workers. However, much less support for the rationing hypothesis
is offered in sections 3 and 4 of Table 4.

Unmeasured taste differences play a more important role in testing
for rationing when the least squares estimates are used to predict desired
hours. As a result, whether or not one interprets the data as supporting
the hypothesis that the unemployed are rationed depends on which model of
estimation one believes is appropriate. Clearly, it will be important in
future work to extend the estimation to include fixed effects, so that these
taste differences can be controlled for. 23/

VI. Conclusion

This paper has presented a method of estimating labor supply func-
tions when part of the sample may be rationed. It has shown that important
labor supply parameters will be significantly affected by ignoring the
rationing problem. The paper has also presented some tests for rationing.
The test results based on the maximum likelihood estimates provide
substantial support for the hypothesis that the unemployed are rationed.
However, the test results based on the least squares estimates indicate that
unmeasured taste differences may be quite important in determining the hours
of work of those who claim to be rationed.

23/ I am currently working on this extension.
### Table 4

**Prediction Performance of Least Squares Estimates (Full Sample) of the Labor Supply Function**

<table>
<thead>
<tr>
<th>Group</th>
<th>Number (n)</th>
<th>Proportion with $\hat{\alpha} &gt; \bar{H}$ ($P_1$)</th>
<th>Mean $\bar{H}$ (in hours)</th>
<th>Mean $\bar{H}$ (in hours)</th>
<th>Mean $\bar{H}$ (in hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 1970 - Entire Sample</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overemployed</td>
<td>52</td>
<td>.288 (0.062)</td>
<td>-211.8</td>
<td>-.121</td>
<td>-54.9</td>
</tr>
<tr>
<td>Under. &amp; not Unem.</td>
<td>122</td>
<td>.680 (0.062)</td>
<td>74.4</td>
<td>.273</td>
<td>231.3</td>
</tr>
<tr>
<td>Unem. &amp; not Under.</td>
<td>66</td>
<td>.563 (0.062)</td>
<td>156.4</td>
<td>.151</td>
<td>313.3</td>
</tr>
<tr>
<td>Unem. &amp; Under.</td>
<td>48</td>
<td>.833 (0.062)</td>
<td>395.5</td>
<td>.421</td>
<td>552.3</td>
</tr>
<tr>
<td>Not Rationed</td>
<td>567</td>
<td>.409 (0.062)</td>
<td>-156.9</td>
<td>.058</td>
<td>88.9</td>
</tr>
<tr>
<td>2. 1972 - Entire Sample</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overemployed</td>
<td>58</td>
<td>.310 (0.062)</td>
<td>-274.8</td>
<td>-.010</td>
<td>-60.4</td>
</tr>
<tr>
<td>Under. &amp; not Unem.</td>
<td>103</td>
<td>.478 (0.062)</td>
<td>36.8</td>
<td>.155</td>
<td>177.5</td>
</tr>
<tr>
<td>Unem. &amp; not Under.</td>
<td>50</td>
<td>.540 (0.062)</td>
<td>116.3</td>
<td>.219</td>
<td>330.7</td>
</tr>
<tr>
<td>Unem. &amp; Under.</td>
<td>18</td>
<td>.778 (0.062)</td>
<td>317.9</td>
<td>.457</td>
<td>532.3</td>
</tr>
<tr>
<td>Not Rationed</td>
<td>627</td>
<td>.321 (0.062)</td>
<td>-241.4</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>3. 1970 - Unemployed or Underemployed in 1971 (R71)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Under. &amp; not Unem.</td>
<td>83</td>
<td>.663 (0.062)</td>
<td>88.3</td>
<td>.178</td>
<td>179.5</td>
</tr>
<tr>
<td>Unem. &amp; not Under.</td>
<td>28</td>
<td>.536 (0.062)</td>
<td>115.4</td>
<td>.051</td>
<td>206.6</td>
</tr>
<tr>
<td>Undem. &amp; Under.</td>
<td>36</td>
<td>.889 (0.062)</td>
<td>443.0</td>
<td>.405</td>
<td>534.2</td>
</tr>
<tr>
<td>Not Rationed</td>
<td>64</td>
<td>.484 (0.062)</td>
<td>-91.2</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>4. 1972 - Unemployed or Underemployed in 1971 (R71)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Under. &amp; not Unem.</td>
<td>60</td>
<td>.467 (0.062)</td>
<td>-57.2</td>
<td>.043</td>
<td>49.4</td>
</tr>
<tr>
<td>Unem. &amp; not Under.</td>
<td>29</td>
<td>.483 (0.062)</td>
<td>119.8</td>
<td>.059</td>
<td>226.4</td>
</tr>
<tr>
<td>Unem. &amp; Under.</td>
<td>13</td>
<td>.638 (0.062)</td>
<td>267.4</td>
<td>.269</td>
<td>373.2</td>
</tr>
<tr>
<td>Not Rationed</td>
<td>111</td>
<td>.423 (0.062)</td>
<td>-106.6</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

$\alpha$ refers to predicted hours while $H$ refers to actual hours.
Excludes workers unemployed and underemployed in same year.
Under. refers to underemployed while Unem. refers to unemployed.
Standard errors in parenthesis.
The paper offers many possible options for future research. First, the model could be extended to include family labor supply considerations. Second, future work should eliminate the restrictive assumption of the exogeneity of the hours worked by the rationed. Third, the estimation technique should be extended to a time-series cross-section framework since the Michigan survey offers longitudinal data.
References


